ADAMA SCIENCE AND TECHNOLOGY UNIVERSITY

SCHOOL OF APPLIED NATURAL SCIENCE

DEPARTMENT OF APPLIED PHYSICS

GENERAL PHYSICS LECTURE NOTE

2022

Contents

Vector

Vectors: composition and resolution

A scalar is a quantity that is completely specified by a number and unit. It has magnitude but no direction. Scalars obey the rules of ordinary algebra. Examples: mass, time, volume, speed, etc.

A vector is a quantity that is specified by both a magnitude and direction in space. Vectors obey the laws of vector algebra. Examples are: displacement, velocity, acceleration, force, torque, momentum, etc.

Vector Representation

Algebraic Method

Vectors are represented algebraically by a letter (or symbol) with an arrow over its head (Example: velocity by $\vec{v}$, momentum by $\vec{P}$) and the magnitude of a vector is a positive scalar and is written as either by |A| or $\vec{A}$.

Geometric Method

Using graphs or line with arrow to represent vectors can be termed as geometric method of representation.

Vector Addition

A single vector that is obtained by adding two or more vectors is called resultant vector and it is obtained using the following two methods

Graphical method of vector addition

Graphically vectors can be added by joining their head to tail and in any order their resultant vector is the vector drawn from the tail of the first vector to the head of the last vector. In Figure 1 graphical technique of vector addition is applied to add three vectors. The resultant vector **R = A + B + C** is the vector that completes the polygon. In other words, R is the vector drawn from the tail of the first vector to the tip of the last vector.

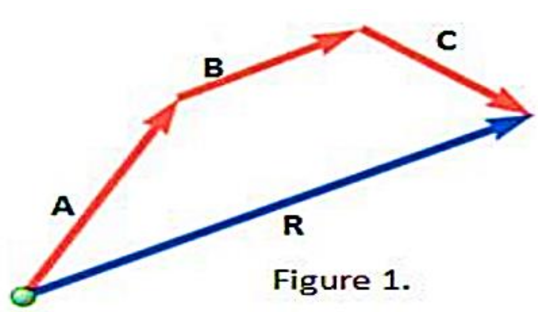


Figure 1.

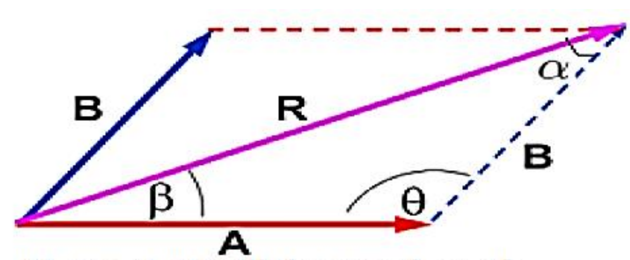


Figure 2. Parallelogram law of vector addition

The parallelogram law states that the resultant R of two vectors A and B is the diagonal of the parallelogram for which the two vectors A and B becomes adjacent sides.

The magnitude of the diagonal (resultant vector) is obtained using cosine law and direction (i.e. the angle that the diagonal vector makes with the sides) is obtained using the sine law.

$$\textbf{Cosine law: } R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

$$\textbf{Sine law: } \frac{\sin\theta}{R} = \frac{\sin\alpha}{A} = \frac{\sin\beta}{B}$$

Components of Vector

Considering Figure 3 below, components of the given vector A are obtained by applying the trigonometric functions of sine and cosine.

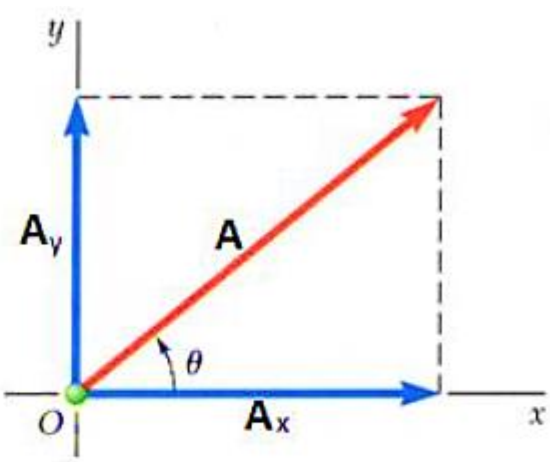


Figure 3: Components of vector **A**

$$\cos\theta = \frac{A_x}{A} \Longrightarrow A_x = A\cos\theta \text{ .... is the x component of } \mathbf{A}$$

$$\sin\theta = \frac{A_y}{A} \Longrightarrow A_y = A\sin\theta \text{ .... Is the y component of } \mathbf{A}$$

Because A_x and A_y are perpendicular to each other, the magnitude of their resultant vector is obtained using Pythagoras theorem.

$$A = \sqrt{A_x^2 + A_y^2}$$

Vector addition in Unit Vector Notation

Unit vector

A unit vector is a vector that has magnitude of one and it is dimensionless and a sole purpose of unit vector is to specify a direction. It is usually denoted with a "hat". ($\hat{u}$)

Consider two unit vectors A and B, then A and B can be added component wise as shown below.

Let $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$ and $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} + (A_z + B_z)\hat{k}$$

Questions and Problems

1. Vector **A** has magnitude of 8units and makes an angle of 45^0 with the positive x-axis. Vector **B** also has the same magnitude of 8units and directed along the negative x-axis. Find

   a. The magnitude and direction of $\vec{A} + \vec{B}$

   b. The magnitude and direction of $\vec{A} - \vec{B}$

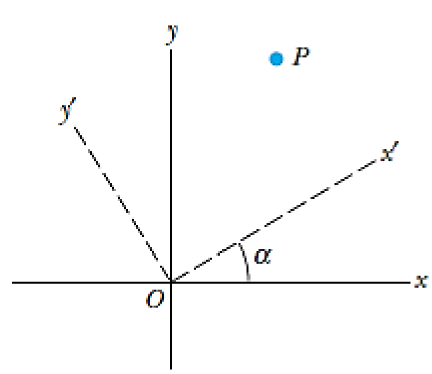
2. Given the displacement vectors $\vec{A} = 3i - 4j + 4k$ and $\vec{B} = 2i + 3j - 7k$. Find the magnitudes of the vectors: (a) $A + B$ (b) $2A - B$

3. If $A = 6i = 8j$ $B = -8i + 3j$ and $C = 26i + 19j$, then find a and B such that $aA + bB + C = 0$.

4. A point P is described by the coordinates (x, y) with respect to the normal Cartesian coordinate system shown in Fig. below. Show that (x', y') the coordinates of this point in the rotated coordinate system, are related to (x, y) and the rotation angle α by the expressions

$$x' = x\cos\alpha + y\sin\alpha$$

$$y' = -x\sin\alpha + y\cos\alpha$$



Kinematics and Dynamics

Motion in One Dimension

Position, Velocity, and Speed

A particle's position is the location of the particle with respect to a chosen reference point that we can consider to be the origin of a coordinate system.

Displacement

The displacement of a particle is defined as its change in position in some time interval. As the moves from an initial position x_i to a final position x_f, the displacement of the particle is given by:

$$\vec{S} = \Delta x = x_f - x_i$$

Whereas distance is the length of a path followed by a particle and can be denoted by **S**.

Note: - distance is a scalar physical quantity whereas displacement is vector.

Average velocity and average speed

Average velocity

The average velocity of a particle is defined as the particle's displacement divided by the time interval during which that displacement occurs:

$$\text{Average velocity} = \frac{\text{displacement}}{\text{total time}}$$

$$\vec{V}_{ave} = \frac{\Delta x}{\Delta t} = \frac{\vec{S}}{t}$$

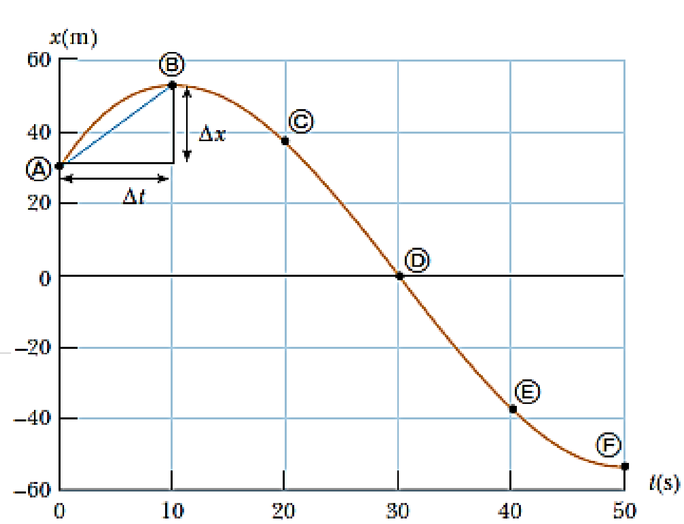
Average speed

The average speed of a particle is defined as the total distance traveled divided by the total time interval required to travel that distance:

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}$$

$$V_{ave} = \frac{x_{total}}{\Delta t} = \frac{S}{t}$$

Example: -



Find the displacement, average velocity, and average speed of the car in the figure below between positions **A** and **F**.

Position	t(s)	x(m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

Solution

Displacement

$$\Delta x = x_F - x_A = -53\text{ m} - 30\text{ m} = -83\text{ m}$$

Average velocity

$$\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{x_F - x_A}{t_F - t_A}$$

$$= \frac{-53\text{ m} - 30\text{ m}}{50\text{ s} - 0\text{ s}} = \frac{-83\text{ m}}{50\text{ s}}$$

$$= -1.7\text{ m/s}$$

Average speed

$$V_{ave} = \frac{x_{total}}{\Delta t} = \frac{127m}{50sec} = 2.5m/sec$$

Instantaneous Velocity and Speed

Instantaneous Velocity

The instantaneous velocity of a particle is defined as the limit of the ratio $\frac{\Delta x}{\Delta t}$ as Δt approaches zero.

$$V_{inst} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

By definition, this limit equals the derivative of x with respect to t, or the time rate of change of the position:

$$V_{inst} = \frac{dx}{dt}$$

The instantaneous speed of a particle is equal to the magnitude of its instantaneous velocity.

Acceleration

Average acceleration

The average acceleration of a particle is defined as the ratio of the change in its velocity ΔV divided by the time interval Δt during which that change occurs:

$$a = \frac{\Delta V}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

Instantaneous acceleration

The instantaneous acceleration is equal to the limit of the ratio $\frac{\Delta V}{\Delta t}$ as Δt approaches 0.

$$a_{inst} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

By definition, this limit equals the derivative of $\vec{v}$ with respect to t, or the time rate of change of the velocity:

$$\vec{a} = \frac{d\vec{V}}{dt}$$

Example: -

The velocity of a particle moving along the x axis varies in time according to the expression $v(t) = (40 - 5t^2)m/s$, where t is in seconds.

(A) Find the average acceleration in the time interval $t = 0s\ ec$ to $t = 2\ s$ec.

Solution:

$$v_i = v(t = 0) = \frac{(40 - 5(0)^2)m}{s} = 40m/s$$

$$v_f = v(t = 2) = \frac{(40 - 5(2)^2)m}{s} = 20m/s$$

$$\vec{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{20m/s - 40m/s}{(2 - 0)sec} = -10m/s^2$$

(B) Determine the acceleration at t = 2 sec.

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d(40 - 5t^2)}{dt} = -10t = -\frac{10(2)m}{s^2} = -20m/s^2$$

One-Dimensional Motion with Constant Acceleration

Equations of motion for one dimensional motion is given by assuming a constant acceleration, hence from

$$a = \frac{v_f - v_i}{t_f - t_i}$$

Using the initial time to be zero

$$v_f = v_i + at \tag{1}$$

Average velocity

$$v_{ave} = \frac{v_i + v_f}{2} \tag{2}$$

Displacement

$$\vec{S} = \Delta x = x_f - x_i = v_{ave} t$$

$$\Delta x = x_f - x_i = \left(\frac{v_i + v_f}{2}\right) t$$

Substitute the value of t

$$\Delta x = x_f - x_i = \left(\frac{v_i + v_f}{2}\right)\left(\frac{v_f - v_i}{a}\right)$$

$$\Delta x = x_f - x_i = \frac{v_f^2 - v_i^2}{2a}$$

$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$

And again using equation (1)

$$\Delta x = x_f - x_i = \left(\frac{v_i + v_f}{2}\right) t$$

Substituting the value of v_f

$$\Delta x = x_f - x_i = \left(\frac{v_i + v_i + at}{2}\right) t$$

$$x_f - x_i = v_i t + \frac{1}{2} a t^2 \tag{3}$$

Example 1

A track covers 40m in 8.5s while smoothly slowing down to a final speed of 2.8m/s. Find

a) Its original speed

b) Its acceleration

Answer:

a) 6.6m/s b) 0.447m/s^2

Example 2

A jet plane lands with a speed of 100m/s and slows down at a rate of 5m/s^2 as it comes to rest.

a) What is the time interval needed by the jet to come to rest?

b) Can this jet land on an airport where the runway is 0.8km long?

Answer: $t = 20 sec$ & $S = 1 km$ hence it cannot land

Exercise

1. At $t = 0s$, a particle moving in the x-y plane with constant acceleration has a velocity of $v_i = (3i - 2j)$ m/s, and is at the origin. At $t = 3s$, the particle‘s velocity is $v_f = (9i + 7j)$ m/s. Find (a) the acceleration of the particle (b) Its coordinates at $t = 3s$
2. Fish swimming in a horizontal plane has velocity $v_i = (4i + 2j)$ m/s at a point in the ocean where the position relative to a certain rock is $r_i = (10i - 4i)m$. After the fish swims with constant acceleration for $20s$, its velocity is $v_f = (20i - 5j)$m/s. Find

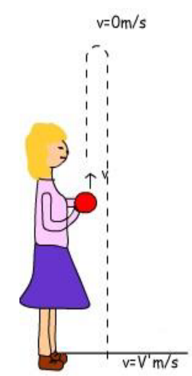
a) The acceleration of the fish

b) If the fish maintains this constant acceleration, determine its position at $t = 25s$?

Free Fall Motion/vertical motion/

The motion of an object near the surface of the Earth under the control of gravitational force is called ***free fall***. In the absence of air resistance, all objects fall with constant acceleration, g towards the surface of the Earth. On the surface of the Earth, the generally accepted value of acceleration due to gravity is 9.8 m/s^2. This acceleration due to gravity varies with latitude, longitude and altitude on the Earth‘s surface. And it is greater at the poles than at the equator and greater at sea level than at the top mountain areas. There are also local variations that depend upon geophysics. The value of 9.8 m/s^2, with only two significant digits, is true for most places on the surface of the Earth up to altitudes of about 16 km.

Example A girl throws a ball upwards, with an initial speed of *v = 15 m/s*. Neglecting air resistance. (a) How long does the ball take to return to the girl‘s hand? (b) What will be its velocity when it reaches at the girl’s hand?



Given: g = -10m/s^2; v_i = 15m/s; $\vec{S} = 0$

a. using the expression:

$$\vec{S} = \Delta y = v_i t - \frac{1}{2} g t^2$$

$$t = 3sec$$

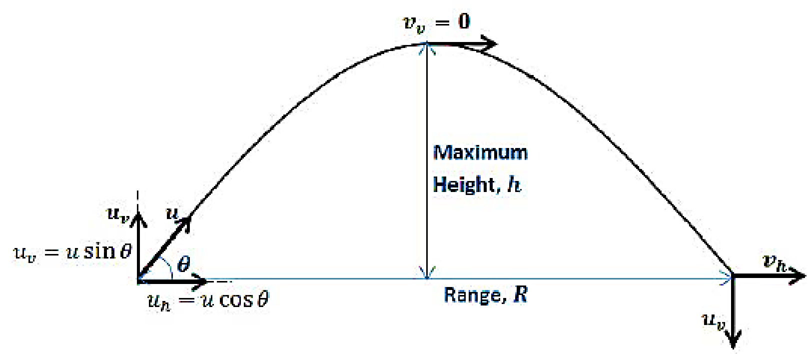
b. $v_f = v_i + gt$

$v_f = 15m/s - 10m/s^2$(3sec)

$$v_f = -15m/s$$

Projectile Motion

Projectile is any object thrown obliquely into the space. The object which is given an initial velocity and afterwards follows a path determined by the gravitational force acting on it is called projectile and the motion is called projectile motion.

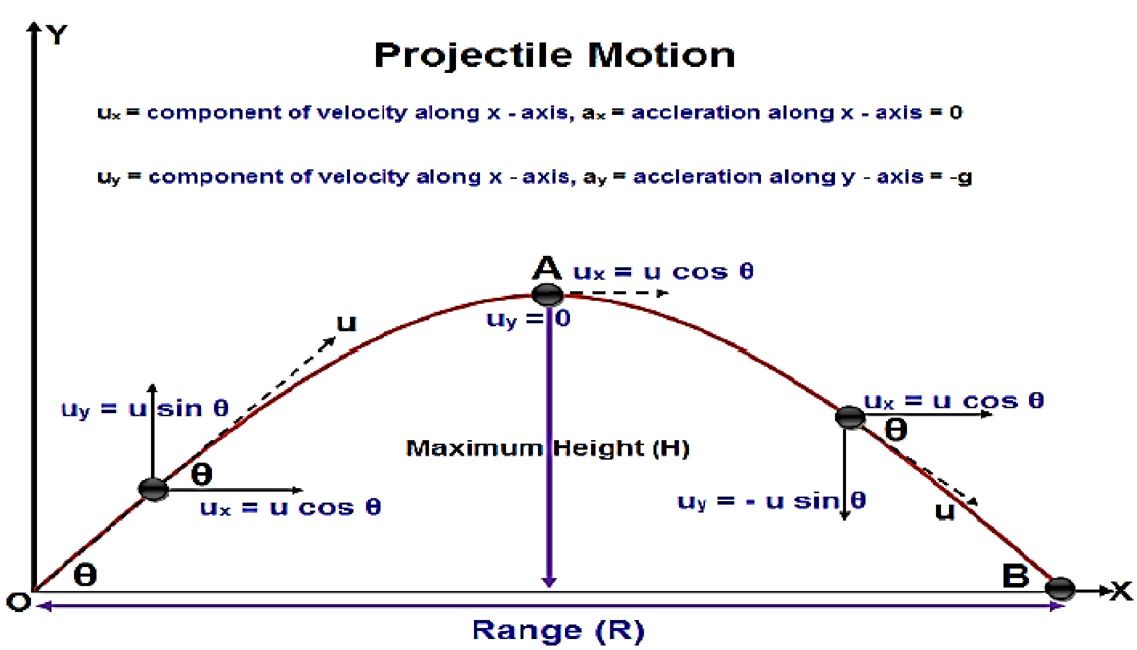


Motion of a projectile

The path described by the projectile from the point of projection to the point where the projectile reaches the horizontal plane passing through the point of projection is called trajectory. The trajectory of the projectile is a ***parabola***.

Basic assumptions in projectile motion

- The free fall acceleration (g) is constant over the range of motion and it is directed downward.
- The effect of air resistance is negligible.



For projectile motion $a_y = -g$ and $a_x = 0$ (Because there is no force acting horizontally)

The horizontal position of the projectile after some time t is:

$$x_f = ucos\theta t + \frac{1}{2}(0)t^2$$

$$x_f = ucos\theta t$$

The vertical position of the projectile after some time t

$$\Delta y = u_y t + \frac{1}{2}a_y t^2$$

$$y_f - usin\theta t - \frac{1}{2}gt^2$$

The horizontal components of the velocity

$$v_x = u_x + a_x t \quad \text{But} a_x = 0$$

$$v_x = u_x - ucos\theta$$

The vertical components of the velocity

$$v_y = u_y + a_y t$$

$$v_y = usin\theta - gt$$

Horizontal Range and Maximum Height

When the projectile reaches the maximum height (the peak), $v_y = 0$

$$0 = usin\theta - gt$$

Hence time to reach maximum height is

$$t = \frac{usin\theta}{g}$$

And the expression for the maximum height will be

$$\text{At } y = h, t = \frac{usin\theta}{g} \rightarrow h = usin\theta\left(\frac{usin\theta}{g}\right) - \frac{1}{2}g\left(\frac{usin\theta}{g}\right)^2$$

$$h = \frac{u^2 sin^2\theta}{2g}$$

The **Range**(R) is the maximum horizontal displacement of the projectile covered in a total time of flight.

$$t_{tot} = 2t, \text{ Where } t = \frac{usin\theta}{g}$$

$$t_{tot} = \frac{2usin\theta}{g}$$

$$\text{When} x = R, t = t_{tot} = \frac{2usin\theta}{g}$$

$$R = (ucos\theta)(\frac{2usin\theta}{g}) \quad \text{But,} \quad 2sin\theta cos\theta = sin2\theta$$

$$R = \frac{u^2 sin2\theta}{g}$$

The range (R) is maximum, when $sin2\theta = 1 \rightarrow 2\theta = 90^0 \rightarrow \theta = 45^0$

$$R_{max} = \frac{u^2}{g}$$

Exercise 1:

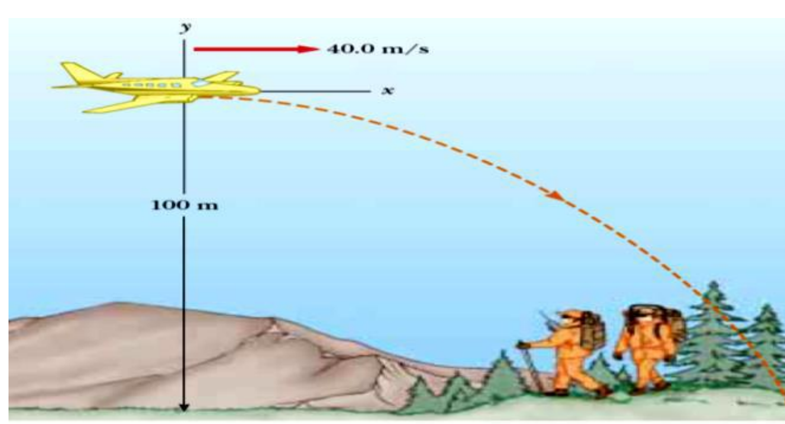
A rocket is fired with an initial velocity of 100m/s at an angle of 55^0 above the horizontal. It explodes on the mountain side 12s after its firing. What is the x-and y- coordinates of the rocket relative to its firing point?

Answer: $x = 688.3m$ and $y = 277m$

Exercise 2:

A plane drops a package to a party of explorer. If the plane is travelling horizontally at 40m/s and is 100m above the ground, where does the package strike the ground relative to the point at which it is released?

Answer: $x = 181m$



Exercise 3

An astronaut on a strange planet can jump a maximum horizontal distance of 15m if his initial speed is 3m/s. What is the free fall acceleration on the planet?

Exercise 4

A ball is thrown with an initial velocity of $(10i + 15j)$ m/s. When it reaches the top of its trajectory what is its velocity?

Exercise 5

A projectile is fired in such a way that its horizontal range is equal to three times its maximum height. What is the angle of projection?

Exercise 6

Two projectiles are thrown with the same initial velocity, one at an angle θ and the other at an angle of $90 - \theta$. (a) Can both projectiles strike the ground at the same distance from the projection point?

(b) Can both projectiles be in air for the same time interval?

Uniform Circular motion

Uniform Circular Motion is motion of objects in a circular path with a constant speed. Objects moving in a circular path with a constant speed can have acceleration.

$$a = \frac{\Delta V}{\Delta t}$$

There are two ways in which the acceleration can occur due to:

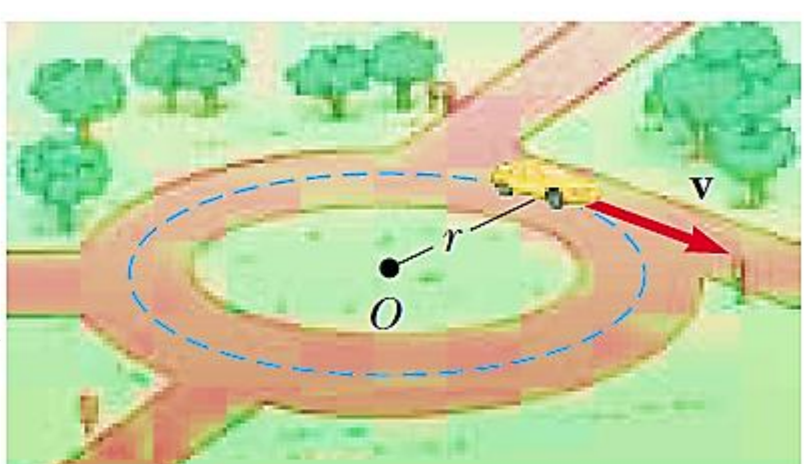
- change in magnitude of the velocity
- change in direction of the velocity

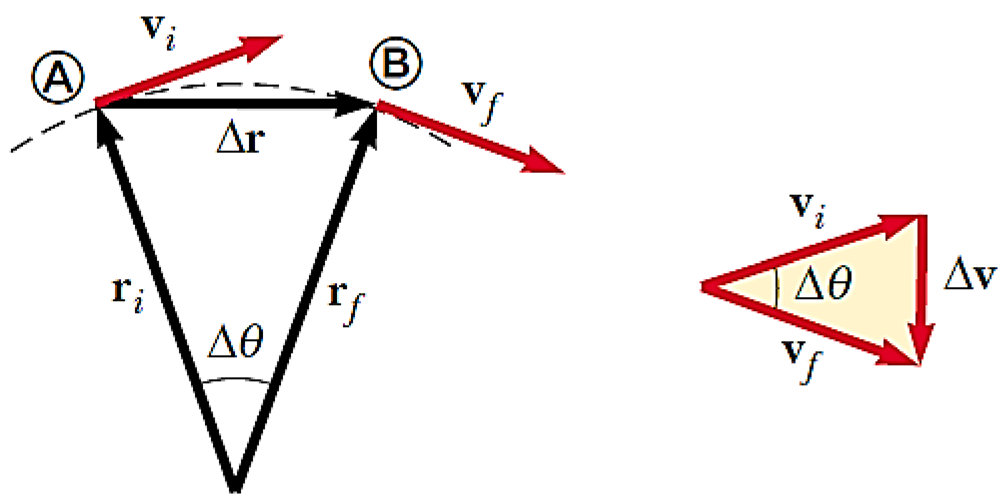
For objects moving in a circular path with a constant speed, acceleration arises because of the change in direction of the velocity.

Hence, in case of uniform circular motion:

- Velocity is always tangent to the circular path and perpendicular to the radius of the circular path.
- Acceleration is always perpendicular to the circular path, and points towards the center of the circle. Such acceleration is called the centripetal acceleration

  A car moving along a circular path at constant speed experiences uniform circular motion.





As a particle moves from **A** to **B**, its velocity vector changes from v_i to v_f .

The average acceleration is defined by the following expression:

$$\overline{\mathbf{a}} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t_f - t_i} = \frac{\Delta \mathbf{v}}{\Delta t}$$

Two triangles are similar if the angle between any two sides is the same for both triangles and if the ratio of the lengths of these sides is the same. Hence this enables us to write a relationship between the lengths of the sides for the two triangles:

Or
$$\frac{|\Delta \mathbf{v}|}{v} = \frac{|\Delta \mathbf{r}|}{r}$$

The ratios of corresponding sides are proportional

Where $v = v_i = v_f$ and $r = r_i = r_f$. This equation can be solved for "Δv" and the expression so obtained can be substituted into to give the magnitude of the average acceleration $a = \frac{\Delta v}{\Delta t}$ over the time interval for the particle to move from **A** to **B**:

In addition, the average acceleration becomes the instantaneous acceleration at point A. Hence, in the limit $\Delta t \rightarrow 0$, the magnitude of the acceleration is

$$a_c = \frac{v^2}{r}$$

Newton's Laws of Motion

Newton's First Law and Inertial Frames

Newton's first law of motion, sometimes called the law of inertia, defines a special set of reference frames called inertial frames. This law can be stated as follows

If an object does not interact with other objects, it is possible to identify a reference frame in which the object has zero acceleration.

Any reference frame that moves with constant velocity relative to an inertial frame is itself an inertial frame.

A more practical statement of Newton's first law of motion:

In the absence of external forces, when viewed from an inertial reference frame, an object at rest remains at rest and an object in motion continues in motion with a constant velocity (that is, with a constant speed in a straight line).

In simpler terms, we can say that ***when no force acts on an object, the acceleration of the object is zero***.

If nothing acts to change the object's motion, then its velocity does not change. From the first law, we conclude that any isolated object (one that does not interact with its environment) is either at rest or moving with constant velocity.

The tendency of an object to resist any attempt to change its velocity is called **inertia**.

Newton's Second Law

When viewed from an inertial reference frame,

The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

$$\sum \mathbf{F} = m\mathbf{a}$$

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z$$

Unit of Force

The SI unit of force is the newton, which is defined as the force that, when acting on an object of mass 1 kg, produces an acceleration of 1 m/s^2.

The Gravitational Force and Weight

The attractive force exerted by the Earth on an object is called the gravitational force F_g. This force is directed toward the center of the Earth,3 and its magnitude is called the weight of the object.

Weight is not an inherent property of an object, but rather a measure of the gravitational force between the object and the Earth. Thus, weight is a property of a system of items—the object and the Earth.
A freely falling object experiences an acceleration g acting toward the center of the Earth. Applying Newton's second law $F = ma$ to a freely falling object of mass m, with a = g and F = F_g , we obtain

$$\mathbf{F}_g = m\mathbf{g}$$

Newton's Third Law

If two objects interact, the force F_{12} exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force F_{21} exerted by object 2 on object 1:

$$\mathbf{F}_{12} = -\mathbf{F}_{21}$$

The forces that object 1 exerts on object 2 may be called the action force and the force of object 2 on object 1 the reaction force. In reality, either force can be labeled the action or reaction force.

- Forces always occur in pairs, or that a single isolated force cannot exist.
- The action force is equal in magnitude to the reaction force and opposite in direction.
- In all cases, the action and reaction forces act on different objects and must be of the same type.

Equilibrium

If the acceleration of an object that can be modeled as a particle is zero, the particle is in equilibrium.
Conditions of equilibrium can also be stated or written as:

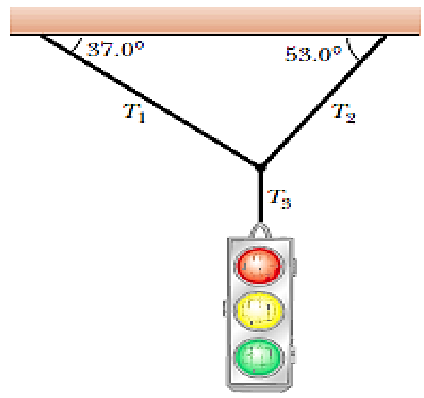
$$\sum F = 0$$

$$\sum F_x = 0$$

$$\sum F_y = 0$$

Example: - 1

A traffic light weighing 122 N hangs from a cable tied to two other cables fastened to a support, as in Figure below. The upper cables make angles of 37° and 53° with the horizontal. These upper cables are not as strong as the vertical cable, and will break if the tension in them exceeds 100 N. Will the traffic light remain hanging in this situation, or will one of the cables break?

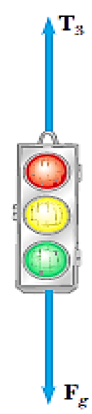


Solution

Assuming that the traffic light is in equilibrium we can draw the free body diagram as

$$\sum F_y = 0$$

$$T_3 = F_g = 122\ \text{N}.$$



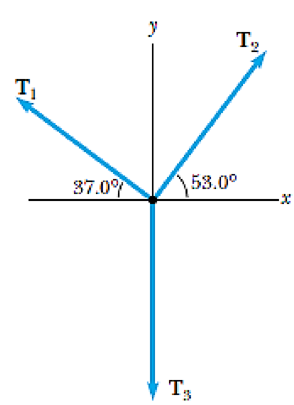
Traffic light suspended by cables

$$\sum F_x = 0$$

$$\sum F_x = -T_1 \cos 37.0° + T_2 \cos 53.0° = 0$$

$$\sum F_y = 0$$

$$\sum F_y = T_1 \sin 37.0° + T_2 \sin 53.0° + (-122\ \text{N}) = 0$$



After combining and substituting the above equations we get

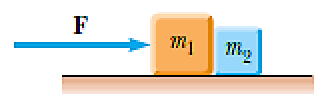
$$T_1 = 73.4 \text{ N}$$

$$T_2 = 1.33T_1 = 97.4 \text{ N}$$

Both of these values are less than 100 N, so the cables will not break

Example: - 2

Two blocks of masses m_1 and m_2, with $m_1 + m_2$, are placed in contact with each other on a frictionless, horizontal surface, as in Figure below. A constant horizontal force F is applied to m_1 as shown. (A) Find the magnitude of the acceleration of the system. (B) Determine the magnitude of the contact force between the two blocks.

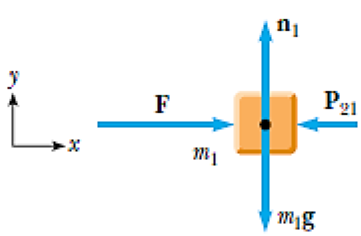


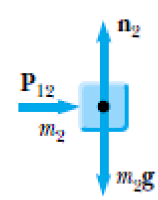
$$\sum F_x(\text{system}) = F = (m_1 + m_2)a_x$$

Solution

$$a_x = \frac{F}{m_1 + m_2}$$

The free body diagrams for the forces are given below





$$\sum F_x = F - P_{21} = F - P_{12} = m_1 a_x$$

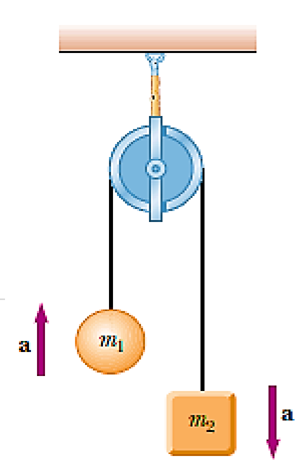
$$P_{12} = F - m_1 a_x = F - m_1\left(\frac{F}{m_1 + m_2}\right) = \left(\frac{m_2}{m_1 + m_2}\right)F$$

$$\sum F_x = P_{12} = m_2 a_x$$

$$m_2 a_x = \left(\frac{m_2}{m_1 + m_2}\right)F$$

P_{12} is the contact force exerted by m_1 on m_2, and P_{21} is the contact force exerted by m_2 on m_1

Example:-3 the Atwood Machine



When two objects of unequal mass are hung vertically over a frictionless pulley of negligible mass, as in Figure below, the arrangement is called an *Atwood machine*. The device is sometimes used in the laboratory to measure the free-fall acceleration. Determine the magnitude of the acceleration of the two objects and the tension in the lightweight cord./$m_2 > m_1$/

Free-body diagrams for the two objects

When Newton's second law is applied to object 1& 2, we obtain

$$\sum F_y = T - m_1 g = m_1 a_y$$

$$\sum F_y = m_2 g - T = m_2 a_y$$

Combining the above equations we get

$$a_y = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g$$

$$T = \left(\frac{2 m_1 m_2}{m_1 + m_2}\right) g$$

Frictional force

When an object is in motion either on a surface or in a viscous medium such as air or water, there is resistance to the motion because the object interacts with its surroundings. We call such resistance a force of friction. Forces of friction are very important in our everyday lives. They allow us to walk or run and are necessary for the motion of wheeled vehicles.

The maximum force of static friction between an object and a surface is proportional to the normal force acting on the object. In general,

$$F_{sf} \leq \mu_s F_n$$

Where, μ_s is the coefficient of static friction and F_n is the magnitude of the normal force. When an object slides over a surface, the direction of the force of kinetic friction F_{kf} is opposite the direction of motion of the object relative to the surface and is also proportional to the magnitude of the normal force. The magnitude of this force is given by

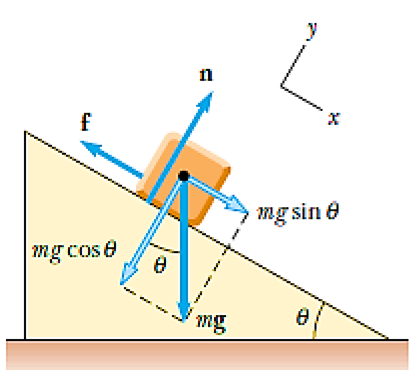
$$F_{kf} = \mu_k F_n$$

, where μ_k is the coefficient of kinetic friction.

Example:-4 Experimental Determination of μ_k and μ_s

The following is a simple method of measuring coefficients of friction: Suppose a block is placed on a rough surface inclined relative to the horizontal, as shown in Figure below.

The incline angle is increased until the block starts to move. Show that by measuring the critical angle θ_c at which this slipping just occurs, we can obtain μ_s .



Newton's second law applied to the block for this balanced situation gives

$$\sum F_x = mg \sin\theta - f_s = ma_x = 0$$

$$\sum F_y = n - mg\cos\theta = ma_y = 0$$

$$\mu_s = \tan\theta_c$$

Example 5 the Sliding Hockey Puck

A hockey puck on a frozen pond is given an initial speed of 20m/s. If the puck always remains on the ice and slides 115 m before coming

$$\sum F_x = -f_k = ma_x$$

$$\sum F_y = n - mg = 0 \qquad (a_y = 0)$$

$$-\mu_k n = -\mu_k mg = ma_x$$

$$a_x = -\mu_k g$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i), \text{ with } x_i = 0 \text{ and}$$

$$0 = v_{xi}^2 + 2a_x x_f = v_{xi}^2 - 2\mu_k g x_f$$

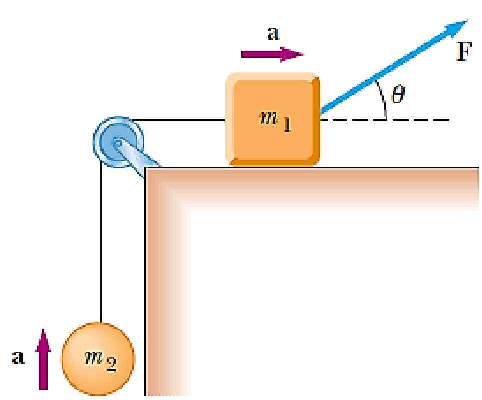
$$\mu_k = \frac{v_{xi}^2}{2gx_f}$$

$$\mu_k = \frac{(20.0\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)(115\ \text{m})} = 0.117$$

to rest, determine the coefficient of kinetic friction between the puck and ice.

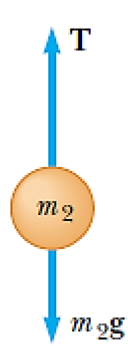
Example 6 Acceleration of Two Connected Objects When Friction Is Present

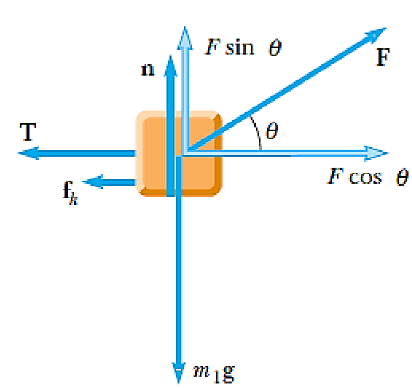
A block of mass m_1 on a rough, horizontal surface is connected to a ball of mass m_2 by a light weight cord over a light weight, frictionless pulley, as shown in Figure below. A force of magnitude F at an angle θ with the horizontal is applied to the block as shown. The coefficient of kinetic friction between the block and surface is μ_k . Determine the magnitude of the acceleration of the two objects.



Solution

The free-body diagrams assuming that the block accelerates to the right and the ball accelerates upward.





Applying Newton's second law to both objects and assuming the motion of the block is to the right, we obtain

$$\sum F_x = F\cos\theta - f_k - T = m_1 a_x = m_1 a$$

$$\sum F_y = T - m_2 g = m_2 a_y = m_2 a$$

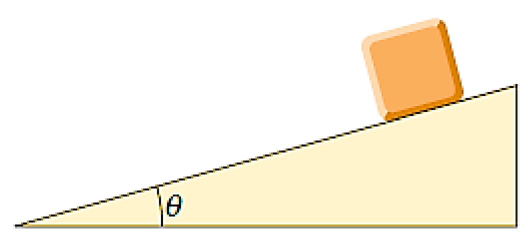
$$\sum F_y = n + F\sin\theta - m_1 g = m_1 a_y = 0$$

Solving for a, we obtain

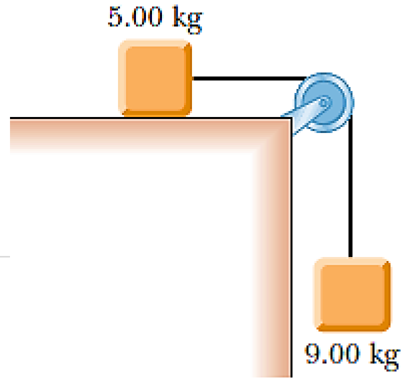
$$a = \frac{F(\cos\theta + \mu_k \sin\theta) - g(m_2 + \mu_k m_1)}{m_1 + m_2}$$

Exercise

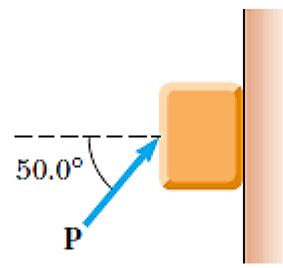
1. A 3kg object undergoes an acceleration given by a = (2i + 5j) m/s^2. Find the resultant force acting on it and the magnitude of the resultant force.
2. If a man weighs 900 N on the Earth, what would he weigh on Jupiter, where the acceleration due to gravity is 25.9 m/s^2?
3. A block is given an initial velocity of 5m/s up a frictionless 20° incline. How far up the incline does the block slide before coming to rest?



4. A 5kg object placed on a frictionless, horizontal table is connected to a string that passes over a pulley and then is fastened to a hanging 9kg object, as in Figure below. Draw free-body diagrams of both objects. Find the acceleration of the two objects and the tension in the string.



5. A block of mass 3kg is pushed up against a wall by a force **P** that makes a 50° angle with the horizontal as shown in Figure below. The coefficient of static friction between the block and the wall is 0.25. Determine the possible values for the magnitude of **P** that allow the block to remain stationary.



Newton's Second Law Applied to Uniform Circular Motion

We discussed that a particle in uniform circular motion, in which the particle moves with constant speed v in a circular path having a radius r. The particle experiences an acceleration that has a magnitude

$$a_c = \frac{v^2}{r}$$

The acceleration is called centripetal acceleration because a_c is directed toward the center of the circle.

If Newton's second law is applied along the radial direction, the net force causing the centripetal acceleration can be related to the acceleration as follows:

$$\sum F = ma_c = m\frac{v^2}{r}$$

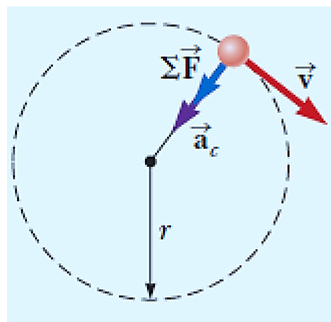
Note:

A force causing a centripetal acceleration acts toward the center of the circular path and causes a change in the direction of the velocity vector. If that force should vanish, the object would no longer move in its circular path; instead, it would move along a straight-line path tangent to the circle.

Analysis Model

Imagine a moving object that can be modeled as a particle. If it moves in a circular path of radius r at a constant speed v, it experiences a centripetal acceleration. Because the particle is accelerating, there must be net force acting on the particle. That force is directed toward the center of the circular path and is given by

$$\sum F = ma_c = m\frac{v^2}{r}$$

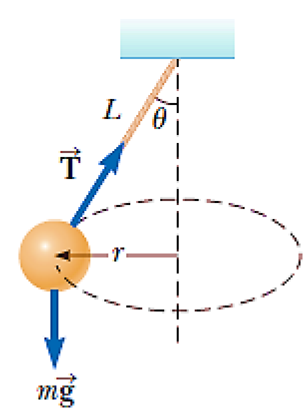


Examples

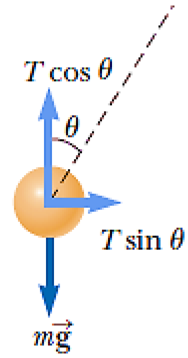
- the tension in a string of constant length acting on a rock twirled in a circle
- the gravitational force acting on a planet traveling around the Sun in a perfectly circular orbit
- the magnetic force acting on a charged particle moving in a uniform magnetic field
- the electric force acting on an electron in orbit around a nucleus in the Bohr model of the hydrogen atom

Example 7 the Conical Pendulum

A small ball of mass m is suspended from a string of length L. The ball revolves with constant speed v in a horizontal circle of radius r as shown in Figure below. Because the string sweeps out the surface of a cone, the system is known as a conical pendulum. Find an expression for v.



The free body diagram for the conical pendulum given above is



We model it as a particle in equilibrium in the vertical direction.

$$\sum F_y = T\cos\theta - mg = 0$$

$$T\cos\theta = mg$$

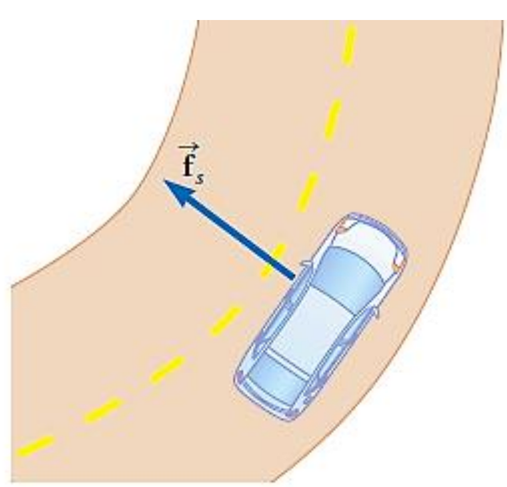
It experiences a centripetal acceleration in the horizontal direction, so it is modeled as a particle in uniform circular motion in this direction.

$$\sum F_x = T\sin\theta = ma_c = \frac{mv^2}{r}$$

$$v = \sqrt{rg\tan\theta}$$

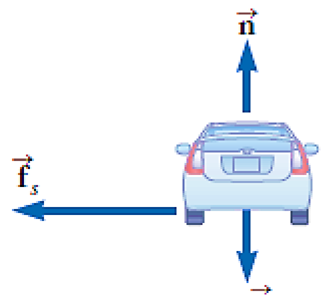
Example 8 What Is the Maximum Speed of the Car?

A 1 500-kg car moving on a flat, horizontal road negotiates a curve as shown in Figure below. If the radius of the curve is 35m and the coefficient of static friction between the tires and dry pavement is 0.523, find the maximum speed the car can have and still make the turn successfully.



Solution

The curved roadway is part of a large circle so that the car is moving in a circular path. We model the car as a particle in uniform circular motion in the horizontal direction.



$$f_{s,\max} = \mu_s n = m\frac{v_{\max}^2}{r}$$

$$\sum F_y = 0 \;\rightarrow\; n - mg = 0 \;\rightarrow\; n = mg$$

$$v_{\max} = \sqrt{\frac{\mu_s n r}{m}} = \sqrt{\frac{\mu_s m g r}{m}} = \sqrt{\mu_s g r}$$

$$v_{\max} = \sqrt{(0.523)(9.80\ \mathrm{m/s^2})(35.0\ \mathrm{m})} = 13.4\ \mathrm{m/s}$$

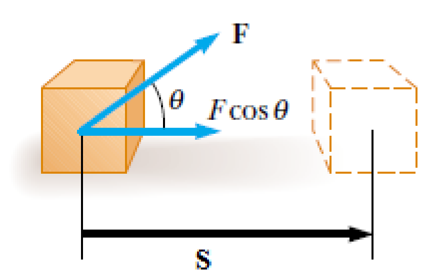
Energy and Energy Transfer

Work Done by a Constant Force

Definition:

Work done by a constant force is defined as the product of the component of the force in the direction of the displacement and the magnitude of the displacement.

Consider the diagram shown below:



If an object undergoes a displacement $\vec{\mathbf{s}}$ under the action of a constant force **F**, the work done by the force is

$$W = (F\cos\theta)\, s = \vec{F}\bullet\vec{s} = \begin{cases} +Fs & \text{if } \theta = 0^\circ \\ 0 & \text{if } \theta = 90^\circ \\ -Fs & \text{if } \theta = 180^\circ \end{cases}$$

Work is a scalar quantity, and its units are force multiplied by length. Therefore, the SI unit of work is the newton meter (N· m). This combination of units is used so frequently that it has been given a name of its own: the joule (J).

Example 1:- A man cleaning a floor pulls a vacuum cleaner with a force of magnitude *F*= 50N at an angle of 30° with the horizontal. Calculate the work done by the force on the vacuum cleaner as the vacuum cleaner is displaced 3m to the right.

Example 2: - A particle moving in the *xy* plane undergoes a displacement r = (2i + 3j) m as a constant force F = (5i + 2j) N acts on the particle. Determine the amount of energy transferred to the particle.

Work Done by a Varying Force

Consider an object that is being displaced along the *x*-axis from x_i to x_f due to the application of a varying positive force *F*(*x*), as shown in the figure (**a**) below. To calculate the work done by this force, we imagine that the object undergoes a very small displacement Δx from x to $x + \Delta x$ due to the effect of an approximate constant force *F*(*x*) as shown in figure (**b**). For this very small displacement, we represent the amount of work done by the force by the expression:

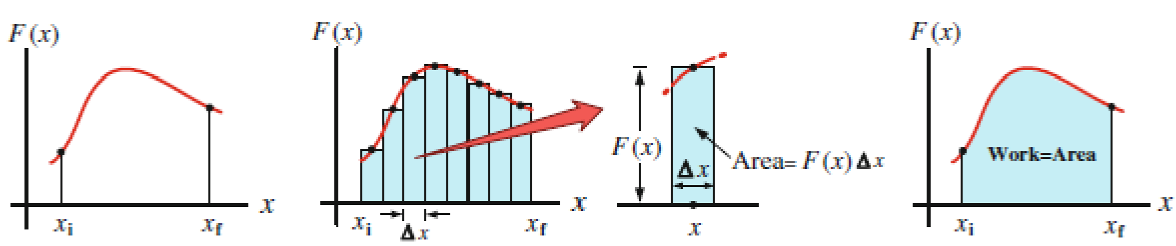
$$\Delta W = F(x)\ \Delta x$$

Then, the total work done from x_i to x_f by the variable force *F*(*x*) is approximately equal to the sum of the large number of rectangles in figure (**b**), i.e. the total area under the force curve. Thus:

$$W \simeq \sum_{x_i}^{x_f} F(x)\ \Delta x$$

In the limit where *x* approaches zero, the value of the sum in the last equation approaches the exact value of the area under the force curve, see figure (c). As you probably know from calculus, the limit of that sum is called an integral and is represented by:

$$\lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F(x)\ \Delta x = \int_{x_i}^{x_f} F(x)\ dx$$



(a) (b) (c)

Therefore, we can express the work done by a variable force $F(x)$ on an object that undergoes a displacement from x_i to x_f as follows:

$$W = \int_{x_i}^{x_f} F(x)\, dx$$

Example: - The force acting on a particle is $F_x = (8x - 16)$ N, where x is in meters. (a) Make a plot of this force versus x from $x = 0$ to $x = 3$m. (b) Find the net work done by this force on the particle as it moves from $x = 0$ to $x = 3$m. **ans: - -12J**

Example: - A force $\mathbf{F} = (4x\mathbf{i} + 3y\mathbf{j})$ N acts on an object as the object moves in the x direction from the origin to $x = 5$m. Find the work done on the object by the force. **ans: - 50J**

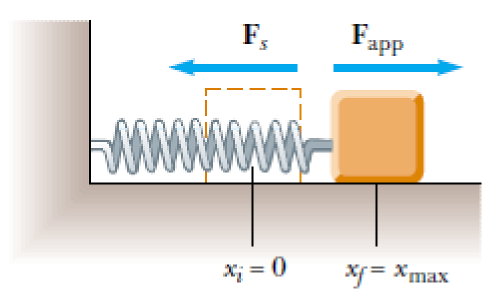
Work done by a spring

A spring is one type of common physical system in which the force (known as the spring force) varies with position. If the spring is stretched or compressed a small distance from equilibrium, the spring will exert a force on the block. This force is given by **Hooke's law** as follows:

$$F = -kx$$

Where $\boldsymbol{x}$ is the displacement of the block from its equilibrium position $(x = 0)$ and $\boldsymbol{k}$ is a positive constant known as the **spring constant** (or the **force constant**).

Consider the following mass spring system in which a block being pulled from $x_i = 0$ to $x_f = x_{max}$ on a frictionless surface by a force $\mathbf{F}_{app}$. If the process is carried out very slowly, the applied force is equal in magnitude and opposite in direction to the spring force at all times.



Therefore, the work done by this applied force (the external agent) on the block–spring system is

$$W_{F_{app}} = \int_0^{x_{max}} F_{app}\, dx = \int_0^{x_{max}} kx\, dx = \tfrac{1}{2}kx_{max}^2$$

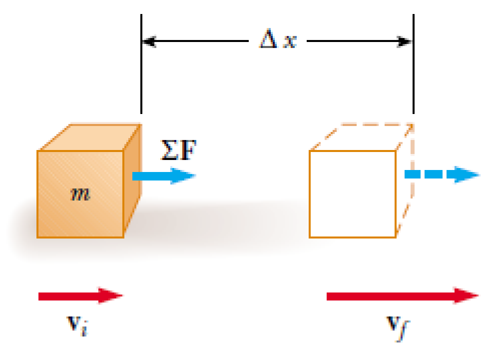
The work done by an applied force on a block–spring system between arbitrary positions of the block is:

$$W_{F_{app}} = \int_{x_i}^{x_f} F_{app}\, dx = \int_{x_i}^{x_f} kx\, dx = \tfrac{1}{2}kx_f^{\,2} - \tfrac{1}{2}kx_i^{\,2}$$

N.B This work is equal to the negative of the work done by the spring force for this displacement.

Kinetic Energy and the Work–Kinetic Energy Theorem

Consider a system consisting of a single object. The figure below shows a block of mass *m* moving through a displacement directed to the right under the action of a net force F, also directed to the right.



Consider a particle of mass *m*, moving with acceleration $a = a(x)$ along the *x*-axis under the effect of a net force $F(x)$ that points along this axis. Thus, according to Newton's second law of motion we have $F(x) = ma(x)$. The work done by this net force on the particle as it moves from an initial position x_i to a final position x_f can be found as follows:

$$W = \int F(x)dx = \int ma(x)dx$$

Using
$$a(x) = \frac{dv}{dt} = v\frac{dv}{dx}$$

$$W = m\int vdv = m\int_{v_i}^{v_f} vdv = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Kinetic Energy

The kinetic energy **K** of a particle is defined as the product of one half of its mass and the square of its speed,

$$K = \frac{1}{2}mv^2$$

Kinetic energy is a scalar quantity and has the same units as work. In SI units we have:

$$1J = 1kg.\frac{m^2}{s^2} = 1N.m$$

Thus, $$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$W_{net} = K_f - K_i = \Delta K$$

$$\mathbf{W_{net} = \Delta K}$$

Is called work-energy theorem which states that "**the work done by the net force in displacing a particle is equal to the change in kinetic energy**."

Change in kinetic energy due to friction can be written as follows:

$$W_{friction} = -F_f.S = \Delta K$$

Example: - A box of mass $m = 10\ kg$ is initially at rest on a rough horizontal surface, where the coefficient of kinetic friction between the box and the surface is $\mu\ = 0.2$. The box is then pulled horizontally by a force F =50N that makes an angle $\theta\ = 60^0$ with the horizontal.

(a) Use the work-energy theorem to find the final speed of the box after it moves a distance of 4m. ***ans: - 3.35m/s***

(b) Repeat part (a) using Newtonian mechanics. ***ans: - 3.35m/s***

Example: - Car traveling at an initial speed ***v*** slides a distance ***d*** to a halt after its brakes lock. Assuming that the car's initial speed is instead **2*v*** at the moment the brakes lock, estimate the distance it slides.

Example: - A block of mass 1.6 kg is attached to a horizontal spring that has a force constant of $1x10^3\ N/m$. The spring is compressed by 2cm and is then released from rest.

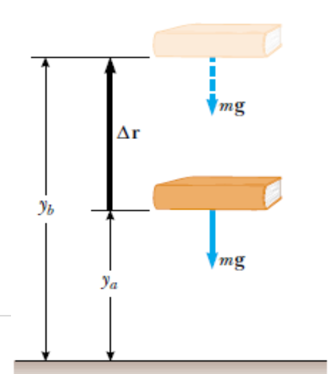
(A) Calculate the speed of the block as it passes through the equilibrium position $x = 0$ if the surface is frictionless. **Ans: - *0.5m/s***

(B) Calculate the speed of the block as it passes through the equilibrium position if a constant friction force of 4N retards its motion from the moment it is released. **Ans: - *0.39m/s***

Potential Energy of a System

Let us imagine a system consisting of a book and the Earth, interacting via the gravitational force.

The work done by an external agent on the system of the book and the Earth as the book is lifted from a height y_a to a height y_b is equal to $mgy_b - mgy_a$



The work done by the external agent on the system (object and Earth) as the object undergoes this upward displacement is given by the product of the upward applied force Fapp and the upward displacement $\Delta\mathbf{r} = \Delta\mathbf{yj}$

$$W = F_{app}.\Delta r = (mgj).(\Delta yj) = mg\Delta y = mg(y_b - y_a)$$

Thus, we can identify the quantity ***mgy*** as the gravitational potential energy ***Ug***

$$U_g = mgy$$

Hence,
$$W = U_b - U_a = U_i - U_f = -\Delta U$$

$$\mathbf{W = -\Delta U}$$

Example: - A 7kg bowling ball held by a careless bowler slips from the bowler's hands and drops on the bowler's toe which is about 0.03m above the floor. Choosing floor level as the $y = 0$ point of your coordinate system, estimate the change in gravitational potential energy of the ball–Earth system as the ball falls from 0.5m height. **Ans: -** $\boldsymbol{-32.24J}$

The Isolated System–Conservation of Mechanical Energy

From work energy theorem and definition of gravitational potential energy

$$W = \Delta K \text{ and } W = -\Delta U$$

Hence,
$$\Delta K + \Delta U = 0$$

$$\Delta(K + U) = 0$$

$$\Delta E = 0$$

Where, $E = K + U$ is called mechanical energy

Conservation of mechanical energy is defined as follows:

From
$$\Delta K + \Delta U = 0$$

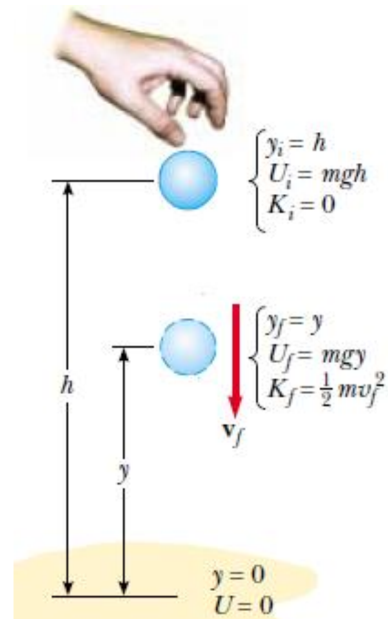
$$K_f - K_i + U_f - U_i = 0$$

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$

Hence, the principle of conservation of mechanical energy states that ***"for an isolated system total mechanical energy of a system is conserved."***

Example: - A ball of mass m is dropped from a height h above the ground, as shown below



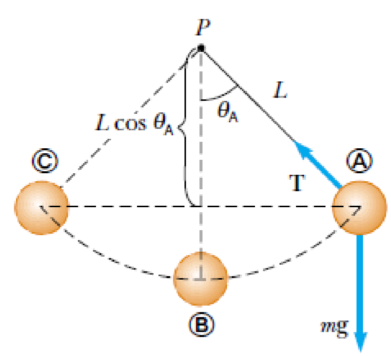
(A) Neglecting air resistance, determine the speed of the ball when it is at a height *y* above the ground.

Ans: -

$$v_f = \sqrt{2g(h - y)}$$

Example: - A pendulum consists of a sphere of mass *m* attached to a light cord of length *L*, as shown in the figure below. The sphere is released from rest at point A when the cord makes an angle *θ* with the vertical, and the pivot at *P* is frictionless.

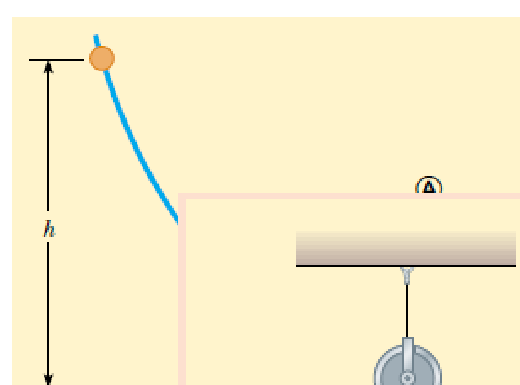
(A) Find the speed of the sphere when it is at the lowest point



Ans: -

$$v_B = \sqrt{2gL(1 - \cos \theta_A)}$$

Example: - A bead slides without friction around a loop-the-loop. The bead is released from a height *h* =3.5*R*. (a) What is its speed at point A? (b) How large is the normal force on it if its mass is 5g?

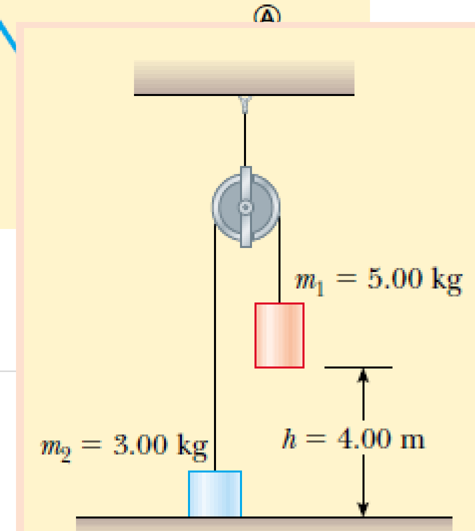


Ans

(a) $v = \sqrt{3gR}$

(b) $F_n = 2mg$

Example: - Two objects are connected by a light string passing over a light frictionless pulley as shown in Figure below.



The object of mass 5kg is released from rest. Using the principle of conservation of energy, (a) determine the speed of the 3kg object just as the 5kg object hits the ground. (b) Find the maximum height to which the 3kg object rises.

Answer: -

(a) 4.43m/s

(b) 5m

Elastic Potential Energy

Previously we learned that the work done by an external applied force F_{app} on a system consisting of a mass connected to the spring is given by:

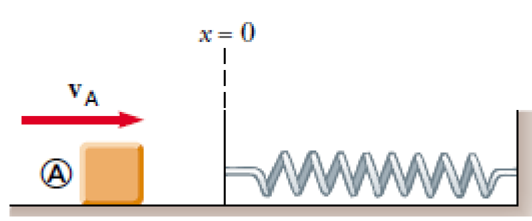
$$W_{F_{\text{app}}} = \int_{x_i}^{x_f} F_{\text{app}}\, dx = \int_{x_i}^{x_f} kx\, dx = \tfrac{1}{2}k{x_f}^2 - \tfrac{1}{2}k{x_i}^2$$

Hence, the ***elastic potential energy*** function associated with the mass–spring system is defined by: -

$$U_s = \frac{1}{2}kx^2$$

The elastic potential energy of the system can be thought of as the energy stored in the deformed spring (one that is either compressed or stretched from its equilibrium position).

Example: - A block having a mass of 0.8kg is given an initial velocity $v_A = 1.2\ m/s$ to the right and collides with a spring of negligible mass and force constant $k = 50\ N/m$, as shown in the figure below.



(A) Assuming the surface to be frictionless, calculate the maximum compression of the spring after the collision. Ans: - ***0.15m***

Conservative and Non-conservative Forces

Conservative Forces

Conservative forces have these two equivalent properties:

1) The work done by a conservative force on a particle moving between any two points is independent of the path taken by the particle.
2) The work done by a conservative force on a particle moving through any closed path is zero. (A closed path is one in which the beginning and end points are identical.)

The gravitational force is one example of a conservative force, and the force that a spring exerts on any object attached to the spring is another

Non-conservative Forces

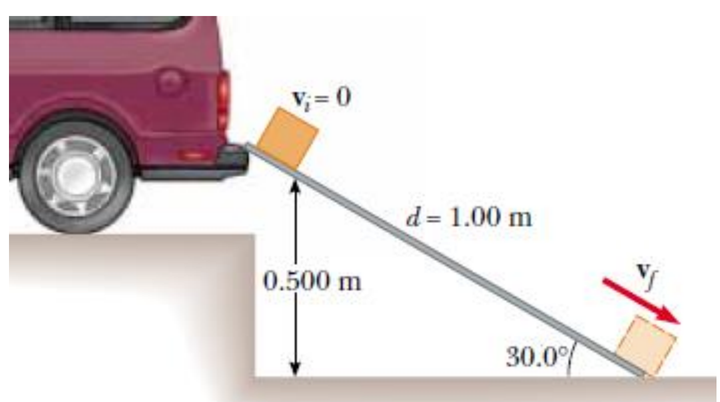
A force is non-conservative if it does not satisfy properties 1 and 2 for conservative forces. Non-conservative forces acting within a system cause a *change* in the mechanical energy of the system. Force of friction is an example of non-conservative force that depends on path hence brings change in mechanical energy or dissipation of energy.

Thus, the change in mechanical energy for non-conservative forces can be written as:

$$\Delta K + \Delta U = W_f = -F_f.d$$

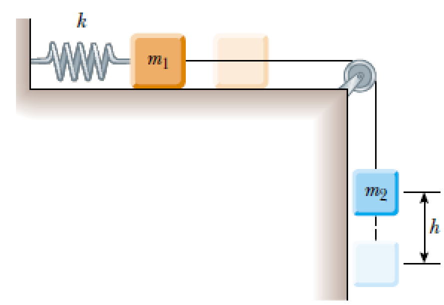
Example: - A 3kg crate slides down a ramp. The ramp is 1m in length and inclined at an angle of 30°, as shown in the figure below. The crate starts from rest at the top, experiences a constant friction force of magnitude 5N, and continues to move a short distance on the horizontal floor after it leaves the ramp. Use energy methods to determine the speed of the crate at the bottom of the ramp.

Ans: - ***2.54m/s***



Example: - A skier starts from rest at the top of a frictionless incline of height 20m, as shown in the figure below. At the bottom of the incline, she encounters a horizontal surface where the coefficient of kinetic friction between the skis and the snow is 0.21. How far does she travel on the horizontal surface before coming to rest, if she simply coasts to a stop? Ans: - ***95.2m***

Example: -Two blocks are connected by a light string that passes over a frictionless pulley, as shown in the figure below. The block of mass m_1 lies on a horizontal surface and is connected to a spring of force constant k. The system is released from rest when the spring is un-stretched. If the hanging block of mass m_2 falls a distance h before coming to rest, calculate the coefficient of kinetic friction between the block of mass m_1 and the surface.



Ans: - $$\mu_k = \frac{m_2 g - \frac{1}{2}kh}{m_1 g}$$

Linear Momentum and Collisions

Consider Newton's second law, when a net force F acts on a particle of mass m, $\vec{F} = m\vec{a}$, which can be generalized for constant and variable mass system as follows:

$$\vec{F} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt}$$

According to this equation, the net force **F** acting on a particle is equal to the change in the product ***mv*** per unit time.

Definition:

The linear momentum of a particle or an object that can be modeled as a particle of mass ***m*** moving with a velocity $\vec{v}$ is defined to be the product of the mass and velocity:

$$\vec{P} = m\vec{v}$$

Linear momentum is a vector quantity and its SI unit is kg · m/s. If a particle is moving in an arbitrary direction, P must have three components,

$$p_x = mv_x \qquad p_y = mv_y \qquad p_z = mv_z$$

Newton's second law for a particle can be written as follows:

$$\sum \mathbf{F} = \frac{d(m\mathbf{v})}{dt} = \frac{d\mathbf{p}}{dt}$$

"The time rate of change of the linear momentum of a particle is equal to the net force acting on the particle."

Principle of conservation of linear momentum

Consider a system of ***n*** particles with linear momenta P_1, P_2, . . . , and P_n. Some forces on these particles are external to the system, and others are internal.

Let *P* be the total linear momentum of the system, which is the vector sum of all individual momenta. Thus:

$$\vec{p}_1 + \vec{p}_2 + \cdots + \vec{p}_n = \sum \vec{p}_i = \vec{P}$$

When differentiating this equation with respect to time, we get:

$$\sum \frac{d\vec{p}_i}{dt} = \sum \vec{F}_i = \frac{d\vec{P}}{dt}$$

Where $\sum F_i$ represents the sum of all *forces* (internal plus external) exerted on the particles of the system. Then

$$\sum \vec{F}_i = \sum \vec{F}_{\text{ext}} + \sum \vec{F}_{\text{int}}$$

By Newton's third law, the internal forces form action-reaction pairs and their sum cancel each other.

$$\sum \vec{F}_{\text{ext}} = \frac{d\vec{P}}{dt}$$

For an isolated system, the sum of the external forces is zero. Hence,

$$\Delta\vec{P} = 0$$

$$\vec{P}_f = \vec{P}_i$$

This is the **law of conservation of momentum** which states that **"**the total linear momentum of an isolated system of particles remains constant.**"**

Fluid Mechanics

Elastic Properties of Solids

All solids are to some extent elastic. This means that we can change their dimensions slightly by pulling, pushing, twisting, and/or compressing them. We shall discuss the elastic properties of solids by introducing the concepts of *stress* and *strain*.

Stress:

Stress is the magnitude of the applied external force that acts perpendicularly on a unit area of the object.

Stress is a quantity that is proportional to the force causing a deformation; more specifically, stress is the external force acting on an object per unit cross-sectional area.

$$\text{Stress} = \frac{F_{\perp}}{A}$$

Strain:

Strain is a measure of the degree of deformation of the object.

Strain in this case is defined as the ratio of the change in length ΔL to the original length L.

$$\text{Strain} = \frac{\Delta L}{L}$$

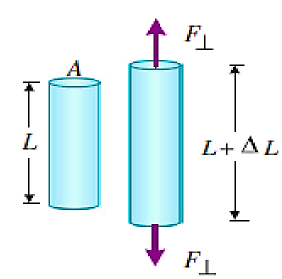


Figure: *A rod of height L and cross-sectional area A, the rod stretches by an amount ΔL after application of a tensile stress.*

It is found that for small stresses, stress is proportional to strain. The *proportionality constant* is called the *elastic modulus* and it depends on the material being deformed, as well as on the nature of the deformation. Therefore:

$$\text{Elastic modulus} = \frac{\text{stress}}{\text{strain}}$$

Three common types of deformation are represented by (1) the resistance of a solid to elongation under a load, characterized by Young's modulus *Y*; (2) the resistance of a solid to the motion of internal planes sliding past each other, characterized by the shear modulus *S*; and (3) the resistance of a solid or fluid to a volume change, characterized by the bulk modulus *B*.

Young's Modulus: Elasticity in Length

Measures the resistance of a solid to a change in its length

Young's Modulus measures the resistance of a solid to a change in its length, which indicates its stiffness.

Consider a metallic long rod of original length L and cross sectional area A. When an external force $F_{\perp}$ is applied perpendicularly to the cross sectional area A of a rod, its internal forces resist its distortion.

$$\text{Tensile stress} = \frac{F_{\perp}}{A} \quad (\text{N/m}^2)$$

$$\text{Tensile strain} = \frac{\Delta L}{L}$$

The relation between the tensile stress and the tensile strain is linear when the rod is in its elastic range. When the stress exceeds what is called the *elastic limit*, the rod is permanently distorted and will not return to its original shape after the stress is removed. As the stress is increased even further, the rod will ultimately break.

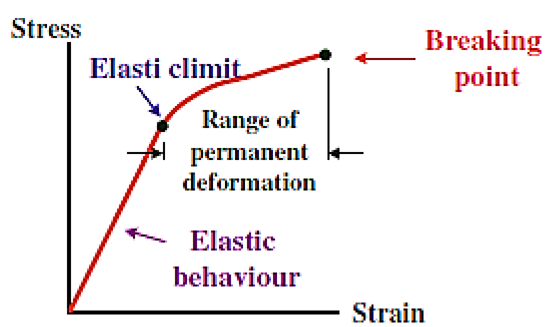
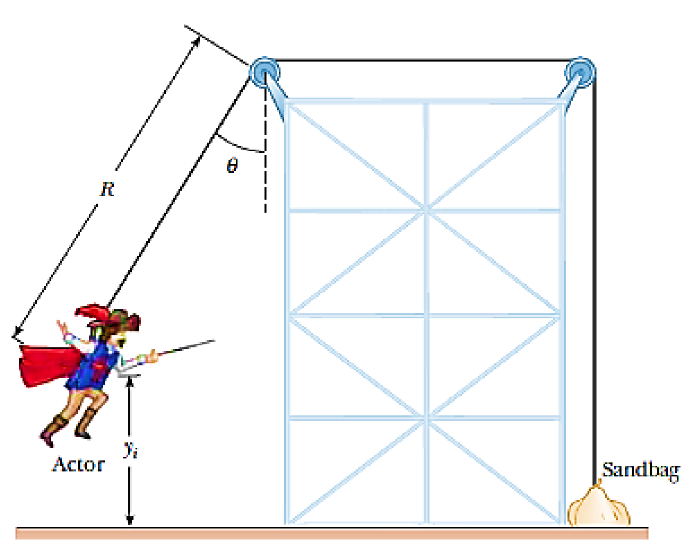


Figure: The stress versus strain curve for an elastic solid

Example: -

Consider a stage design by which a cable is used to support an actor as he swung onto the stage. Suppose that the tension in the cable is 940 N as the actor reaches the lowest point. What diameter should a 10m long steel wire have if we do not want it to stretch more than 0.5 cm under these conditions?



Solution

$$A = \frac{FL_i}{Y\Delta L} = \frac{(940\ \text{N})(10\ \text{m})}{(20 \times 10^{10}\ \text{N/m}^2)(0.005\ \text{m})}$$

$$= 9.4 \times 10^{-6}\ \text{m}^2$$

Because $A = \pi r^2$, the radius of the wire can be found from

$$r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{9.4 \times 10^{-6}\ \text{m}^2}{\pi}} = 1.7 \times 10^{-3}\ \text{m} = 1.7\ \text{mm}$$

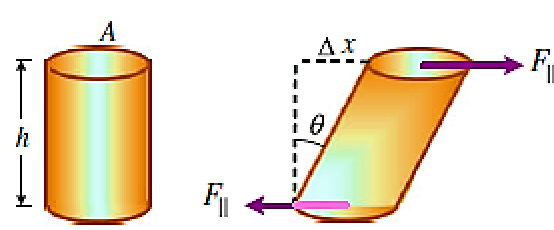
$$d = 2r = 2(1.7\ \text{mm}) = 3.4\ \text{mm}$$

Shear Modulus:

Measures the resistance to motion of the planes of solids when sliding over each other

Another type of deformation occurs when a solid is subject to a force applied parallel to one of its surfaces while the opposite surface is kept fixed.

Figure below shows a cylindrical rod subjected to a linear or torsional shear stress deforming it by an amount $\Delta \boldsymbol{x}$ due to a force $\boldsymbol{F}_{||}$ parallel to the surface area $\boldsymbol{A}$. As a final result, the shape of the rod will attain equilibrium when the effect of the shear force $\boldsymbol{F}_{||}$ balances exactly the internal shear forces.



For linear shearing, we define the *shearing stress* and the *shearing strain* as follows:

$$\text{Shearing stress} = \frac{\text{Tangential acting force}}{\text{Area of surface being sheared}} = \frac{F_{\|}}{A} \quad (\text{N/m}^2)$$

$$\text{Shearing strain} = \frac{\text{Distance sheared}}{\text{Distance between surfaces}} = \frac{\Delta x}{h} = \tan\theta \simeq \theta$$

$$S = \frac{\text{Shearing stress}}{\text{Shearing strain}} = \frac{F_{\|}/A}{\Delta x/h} \quad (\text{N/m}^2)$$

S is also called the modulus of rigidity or the torsion modulus and is significant only for solids.

Example: - A steel cable 3cm^2 in cross-sectional area has a mass of 2.4kg per meter of length. If 500m of the cable is hung over a vertical cliff, how much does the cable stretch under its own weight? $Y_{steel} = 2 \, x \, 10^{11}$ N/m^2.

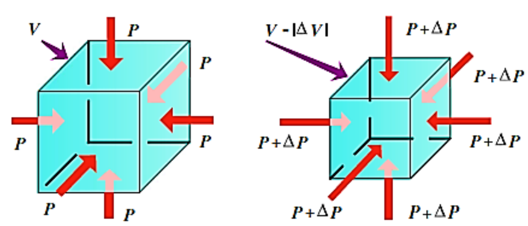
Bulk Modulus:

Measures the resistance of a solid (or a liquid) to a change in its volume

Another type of deformation occurs when an object is subject to an equal increase in normal forces acting on all its faces. For such a study, it is appropriate to define the pressure P as the force acting perpendicularly on a unit area of the object. That is:

$$P = \frac{F_\perp}{A} \quad (\text{N/m}^2)$$

When the force $\boldsymbol{F}$ on each face increases, the pressure will increase too and consequently the volume **V** will decrease as shown below:



Hence

$$\text{Volume stress} = \Delta P = \frac{\Delta F_\perp}{A}$$

$$\text{Volume strain} = -\frac{\Delta V}{V}$$

$$B = \frac{\text{Volume stress}}{\text{Volume strain}} = -\frac{\Delta F_\perp / A}{\Delta V / V} = -\frac{\Delta P}{\Delta V / V} \Rightarrow B = -V\frac{dP}{dV}$$

Typical Values for Elastic Moduli

Substance	Young's Modulus (N/m^2)	Shear Modulus (N/m^2)	Bulk Modulus (N/m^2)
Tungsten	35×10^{10}	14×10^{10}	20×10^{10}
Steel	20×10^{10}	8.4×10^{10}	6×10^{10}
Copper	11×10^{10}	4.2×10^{10}	14×10^{10}
Brass	9.1×10^{10}	3.5×10^{10}	6.1×10^{10}
Aluminum	7.0×10^{10}	2.5×10^{10}	7.0×10^{10}
Glass	$6.5\text{–}7.8 \times 10^{10}$	$2.6\text{–}3.2 \times 10^{10}$	$5.0\text{–}5.5 \times 10^{10}$
Quartz	5.6×10^{10}	2.6×10^{10}	2.7×10^{10}
Water	—	—	0.21×10^{10}
Mercury	—	—	2.8×10^{10}

Example: -

A solid brass sphere is initially surrounded by air, and the air pressure exerted on it is $1 \times 10^5/\text{m}^2$ (normal atmospheric pressure). The sphere is lowered into the ocean to a depth where the pressure is 2×10^7 N/m^2. The volume of the sphere in air is 0.50 m^3. By how much does this volume change once the sphere is submerged?

<u>Solution</u>

$$\Delta V = -\frac{V_i \Delta P}{B}$$

Substituting the numerical values, we obtain

$$\Delta V = -\frac{(0.50\ \text{m}^3)(2.0 \times 10^7\ \text{N/m}^2 - 1.0 \times 10^5\ \text{N/m}^2)}{6.1 \times 10^{10}\ \text{N/m}^2}$$

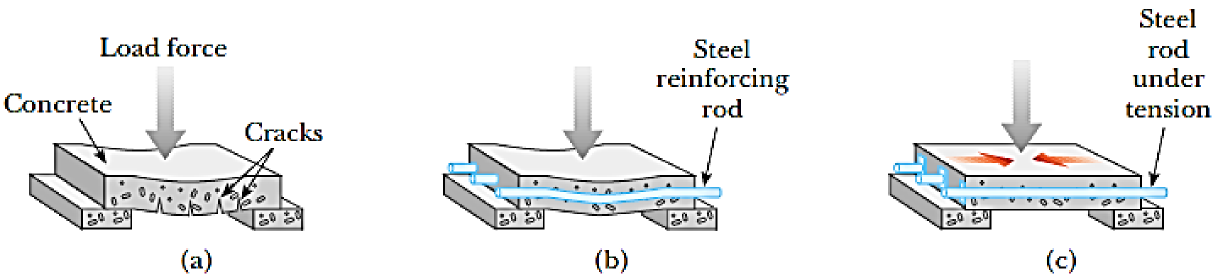
$$= -1.6 \times 10^{-4}\ \text{m}^3$$

The negative sign indicates that the volume of the sphere decreases.

Application

Pre-stressed Concrete

If the stress on a solid object exceeds a certain value, the object fractures. The maximum stress that can be applied before fracture occurs depends on the nature of the material and on the type of applied stress. For example, concrete has a tensile strength of about 2×10^6 *N/m², a compressive strength of* 20×10^6 *N/m², and shear strength of* 2×10^6 *N/m². If the applied stress exceeds these values, the concrete fractures. It is common practice to use large safety factors to prevent failure in concrete structures.*



(a) A concrete slab with no reinforcement tends to crack under a heavy load.

(b) The strength of the concrete is increased by using steel reinforcement rods.

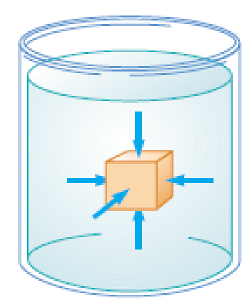
(c) The concrete is further strengthened by pre-stressing it with steel rods under tension.

Fluid mechanics

A **fluid** is a collection of molecules that are randomly arranged and held together by weak cohesive forces and by forces exerted by the walls of a container. Both liquids and gases are fluids.

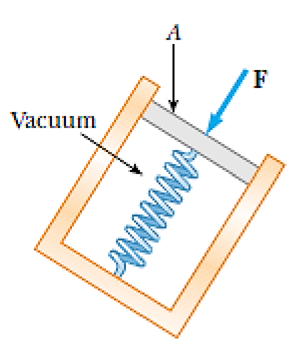
Pressure

Fluids do not sustain shearing stresses or tensile stresses; thus, the only stress that can be exerted on an object submerged in a static fluid is one that tends to compress the object from all sides. In other words, the force exerted by a static fluid on an object is always perpendicular to the surfaces of the object.



At any point on the surface of a submerged object, the force exerted by the fluid is perpendicular to the surface of the object. The force exerted by the fluid on the walls of the container is perpendicular to the walls at all points.

Consider the following simple device used to measure the pressure inside a fluid



If **F** is the magnitude of the force exerted on the piston and **A** is the surface area of the piston, then the pressure P of the fluid at the level to which the device has been submerged is defined as

$$P = \frac{F}{A}$$

The SI unit for pressure is **Pascal**

$$1Pa = 1N/m^2$$

Example: - The mattress of a water bed is 2m long by 2m wide and 3cm deep.

(A) Find the weight of the water in the mattress.

$$W = 1.18 \times 10^4 \text{ N}$$

(B) Find the pressure exerted by the water on the floor when the bed rests in its normal position.

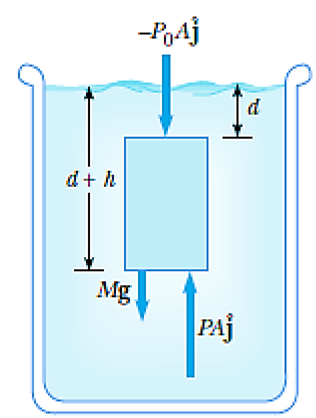
$$2.95 \times 10^3 \text{ Pa}$$

Variation of Pressure with Depth

Now consider a liquid of density ρ at rest as shown in figure below. We assume that ρ is uniform throughout the liquid; this means that the liquid is incompressible.

Let us select a sample of the liquid contained within an imaginary cylinder of cross-sectional area $\boldsymbol{A}$ extending from depth d to depth $d + h$.

The net force exerted on the parcel of fluid must be zero because it is in equilibrium.



$$\sum \mathbf{F} = PA\hat{\mathbf{j}} - P_0A\hat{\mathbf{j}} - M\mathbf{g}\hat{\mathbf{j}} = 0$$

$$PA - P_0A - \rho Ahg = 0$$

$$PA - P_0A = \rho Ahg$$

$$P = P_0 + \rho gh$$

Where,

$$P_0 = 1.00 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$

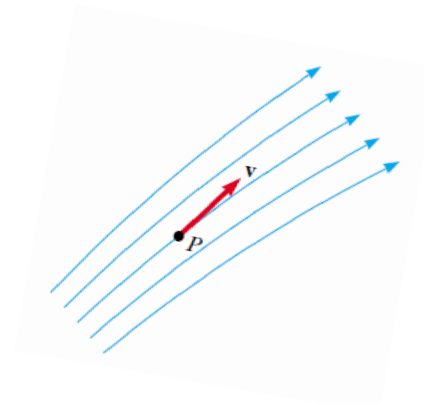
Fluid dynamics

When fluid is in motion, its flow can be characterized as being one of two main types.

Laminar flow

The flow is said to be steady, or ***laminar***, if each particle of the fluid follows a smooth path, such that the paths of different particles never cross each other, In steady flow, the velocity of fluid particles passing any point remains constant in time.

A particle in laminar flow follows a streamline, and at each point along its path the particle's velocity is tangent to the streamline.



Turbulent flow

Turbulent flow is irregular flow characterized by small whirlpool-like regions.

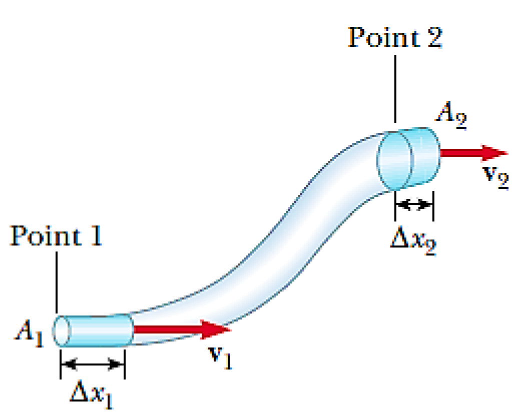
Viscosity

Viscosity is commonly used in the description of fluid flow to characterize the degree of internal friction in the fluid. This internal friction, or viscous force, is associated with the resistance that two adjacent layers of fluid have to moving relative to each other. Viscosity causes part of the kinetic energy of a fluid to be converted to internal energy.

In our model of ideal fluid flow, we make the following four assumptions:

- ✓ ***The fluid is non-viscous***. In a non-viscous fluid, internal friction is neglected. An object moving through the fluid experiences no viscous force.
- ✓ ***The flow is steady***. In steady (laminar) flow, the velocity of the fluid at each point remains constant.
- ✓ ***The fluid is incompressible***. The density of an incompressible fluid is constant.
- ✓ The ***flow is irrotational***. In irrotational flow, the fluid has no angular momentum about any point. If a small paddle wheel placed anywhere in the fluid does not rotate about the wheel's center of mass, then the flow is irrotational.

Consider an ideal fluid flowing through a pipe of non-uniform size



The fluid is incompressible and because the flow is steady, the mass that crosses A_1 in a time interval Δt must equal the mass that crosses A_2 in the same time interval.

$$m_1 = m_2$$

$$\rho_1 V_1 = \rho_2 V_2$$

$$\rho_1 A_1 \Delta x_1 = \rho_2 A_2 \Delta x_2$$

$$\rho_1 A_1 v_1 \Delta t_1 = \rho_2 A_2 v_2 \Delta t_2$$

$$\Delta t_1 = \Delta t_2 = \Delta t$$

$$A_1 v_1 = A_2 v_2$$

This expression is called the equation of continuity for fluids. It states that "the product of the area and the fluid speed at all points along a pipe is constant for an incompressible fluid."

Example: - **1** A water hose 2.5cm in diameter is used by a gardener to fill a 30L bucket. The gardener notes that it takes 1.00 min to fill the bucket. A nozzle with an opening of cross-sectional area 0.5cm^2 is then attached to the hose. The nozzle is held so that water is projected horizontally from a point 1m above the ground. Over what horizontal distance can the water be projected?

Solution

$$A_1 = \pi r^2 = \pi \frac{d^2}{4} = \pi \left(\frac{(2.50\text{ cm})^2}{4} \right) = 4.91\text{ cm}^2$$

According to the data given, the volume flow rate is equal to 30.0 L/min:

$$A_1 v_1 = 30.0\text{ L/min} = \frac{30.0 \times 10^3\text{ cm}^3}{60.0\text{ s}} = 500\text{ cm}^3/\text{s}$$

$$v_1 = \frac{500\text{ cm}^3/\text{s}}{A_1} = \frac{500\text{ cm}^3/\text{s}}{4.91\text{ cm}^2} = 102\text{ cm/s} = 1.02\text{ m/s}$$

$$A_1 v_1 = A_2 v_2 = A_2 v_{xi} \longrightarrow v_{xi} = \frac{A_1}{A_2} v_1$$

$$v_{xi} = \frac{4.91 \text{ cm}^2}{0.500 \text{ cm}^2}(1.02 \text{ m/s})$$
$$= 10.0 \text{ m/s}$$

$$y_f = y_i + v_{yi}t - \tfrac{1}{2}gt^2$$
$$-1.00 \text{ m} = 0 + 0 - \tfrac{1}{2}(9.80 \text{ m/s}^2)t^2$$
$$t = \sqrt{\frac{2(1.00 \text{ m})}{9.80 \text{ m/s}^2}} = 0.452 \text{ s}$$

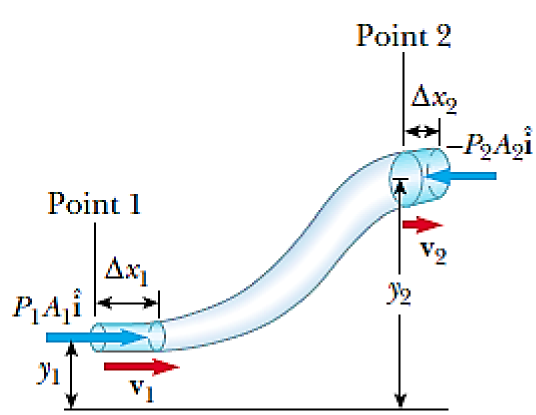
In the horizontal direction, we apply Equation 2.12 with $a_x = 0$ to a particle of water to find the horizontal distance:

$$x_f = x_i + v_{xi}t = 0 + (10.0 \text{ m/s})(0.452 \text{ s}) = 4.52 \text{ m}$$

Bernoulli's Equation

As a fluid moves through a region where its speed and/or elevation above the Earth's surface changes, the pressure in the fluid varies with these changes.

Consider the flow of a segment of an ideal fluid through a non-uniform pipe in a time interval 't, as illustrated in Figure below



The work done by the force on the segment in a time interval Δt is

$$W = (P_1 - P_2)\Delta V$$

Part of this work goes into changing the kinetic energy of the segment of fluid, and part goes into changing the gravitational potential energy of the segment–Earth system.

Thus, the change in the kinetic energy of the segment of fluid is

$$\Delta K = \frac{1}{2}mv_2{}^2 - \frac{1}{2}mv_1{}^2$$

Consequently, the change in gravitational potential energy is

$$\Delta U = mgy_2 - mgy_1$$

The total work done on the system by the fluid outside the segment is equal to the change in mechanical energy of the system:

$$W = \Delta K + \Delta U$$

$$(P_1 - P_2)V = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1{}^2 + mgy_2 - mgy_1$$

$$P_1 - P_2 = \frac{1}{2}\rho v_2{}^2 - \frac{1}{2}\rho v_1^2 + \rho g y_2 - \rho g y_1$$

$$P_1 + \frac{1}{2}\rho v_1{}^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2{}^2 + \rho g y_2$$

This is Bernoulli's equation as applied to an ideal fluid. It is often expressed as

$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

Example: - 1

A horizontal pipe 10cm in diameter has a smooth reduction to a pipe 5cm in diameter. If the pressure of the water in the larger pipe is $8x10^4$ Pa and the pressure in the smaller pipe is $6x\ 10^4$ Pa, at what rate does water flow through the pipes?

Pascal's law states that when pressure is applied to an enclosed fluid, the pressure is transmitted undiminished to every point in the fluid and to every point on the walls of the container.

When an object is partially or fully submerged in a fluid, the fluid exerts on the object an upward force called the buoyant force. According to Archimedes' principle, the magnitude of the buoyant force is equal to the weight of the fluid displaced by the object:

$$B = \rho_{\text{fluid}}\, gV$$

You can understand various aspects of a fluid's dynamics by assuming that the fluid is non-viscous and incompressible, and that the fluid's motion is a steady flow with no rotation.

Two important concepts regarding ideal fluid flow through a pipe of non-uniform size are as follows:

1. The flow rate (volume flux) through the pipe is constant; this is equivalent to stating that the product of the cross-sectional area A and the speed v at any point is a constant. This result is expressed in the equation of continuity for fluids:

$$A_1 v_1 = A_2 v_2 = \text{constant}$$

2. The sum of the pressure, kinetic energy per unit volume, and gravitational potential energy per unit volume has the same value at all points along a streamline. This result is summarized in Bernoulli's equation:

$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

Heat and Thermodynamics

Just read only your exercise books about heat, thermodynamics laws etc

Oscillations and Mechanical Waves

Simple harmonic motion

Motion of an Object Attached to a Spring As a model for simple harmonic motion;

Consider a block of mass m attached to the end of a spring, with the block free to move on a frictionless, horizontal surface

When the spring is neither stretched nor compressed, the block is at rest at the position called the equilibrium position of the system, which we identify as x = 0. We know from experience that such a system oscillates back and forth if disturbed from its equilibrium position.

We can understand the oscillating motion of the block in Figure 15.1 qualitatively by first recalling that when the block is displaced to a position x, the spring exerts on the block a force that is proportional to the position and given by **Hooke's law**

$$F_s = -kx$$

We call F_s a restoring force because it is always directed toward the equilibrium position and therefore opposite the displacement of the block from equilibrium. That is, when the block is displaced to the right of x= 0 in Figure, the position is positive and the restoring force is directed to the left. When the block is displaced to the left of $x = 0$ as in Figure, the position is negative and the restoring force is directed to the right.

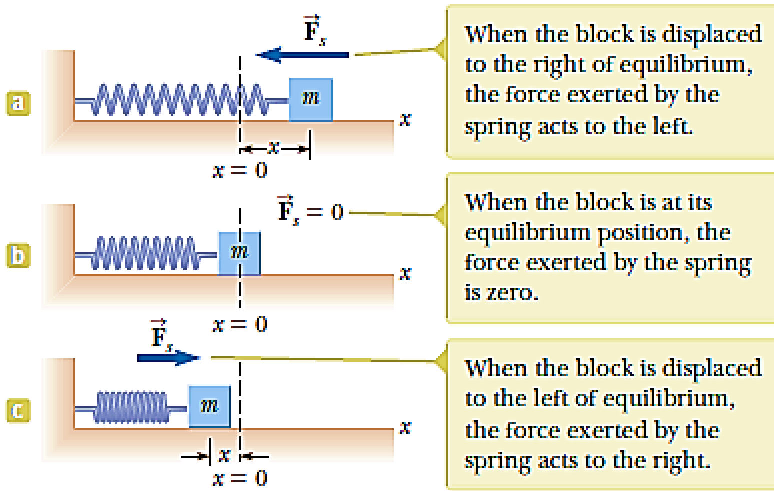
When the block is displaced from the equilibrium point and released, it is a particle under a net force and consequently undergoes acceleration. Applying the particle under a net force model to the motion of the block, with the above providing the net force in the x direction, we obtain

$$\sum F_x = ma_x \rightarrow -kx = ma_x$$

$$a_x = -\frac{k}{m}x$$

That is, the acceleration of the block is proportional to its position, and the direction of the acceleration is opposite the direction of the displacement of the block from equilibrium. Systems that behave in this way are said to exhibit simple harmonic motion. ***An object moves with simple harmonic motion***

whenever its acceleration is proportional to its position and is oppositely directed to the displacement from equilibrium.



Quiz A block on the end of a spring is pulled to position $x = A$ and released from rest. In one full cycle of its motion, through what total distance does it travel? (a) A/2 (b) A (c) 2A (d) 4A

Particle in Simple Harmonic Motion

The motion described in the preceding section occurs so often that we identify the particle in simple harmonic motion model to represent such situations. To develop a mathematical representation for this model, we will generally choose x as the axis along which the oscillation occurs; hence, we will drop the subscript-x notation in this discussion.

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

The above equation can be written in the form

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

The following cosine function is a solution to the differential equation:

$$x(t) = A\cos(\omega t + \phi)$$

To show explicitly that this solution satisfies Equation, notice that

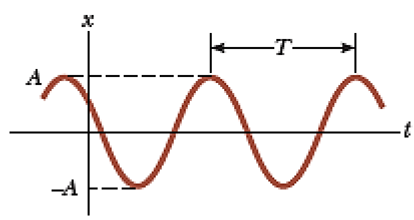
$$\frac{dx}{dt} = A\frac{d}{dt}\cos(\omega t + \phi) = -\omega A \sin(\omega t + \phi)$$

$$\frac{d^2x}{dt^2} = -\omega A\frac{d}{dt}\sin(\omega t + \phi) = -\omega^2 A \cos(\omega t + \phi)$$

A, called the **amplitude** of the motion, is simply the maximum value of the position of the particle in either the positive or negative x direction.

The constant ω is called the **angular frequency**, and it has units of radians per second. It is a measure of how rapidly the oscillations are occurring; the more oscillations per unit time, the higher the value of ω

$$\omega = \sqrt{\frac{k}{m}}$$



An x–t graph for a particle undergoing simple harmonic motion

The period T of the motion is the time interval required for the particle to go through one full cycle of its motion. That is, the values of x and v for the particle at time t equal the values of x and v at time t + T. Because the phase increases by 2π radians in a time interval of T,

$$[\omega(t + T) + \phi] - (\omega t + \phi) = 2\pi$$

Simplifying this expression gives

$$T = \frac{2\pi}{\omega}$$

The inverse of the period is called the frequency f of the motion. Whereas the period is the time interval per oscillation, the frequency represents the number of oscillations the particle undergoes per unit time interval:

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

The units of f are cycles per second, or hertz (Hz).

The period and frequency of the motion for the particle in simple harmonic motion in terms of the characteristics m and k of the system as

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

We can obtain the velocity and acceleration of a particle undergoing simple harmonic motion

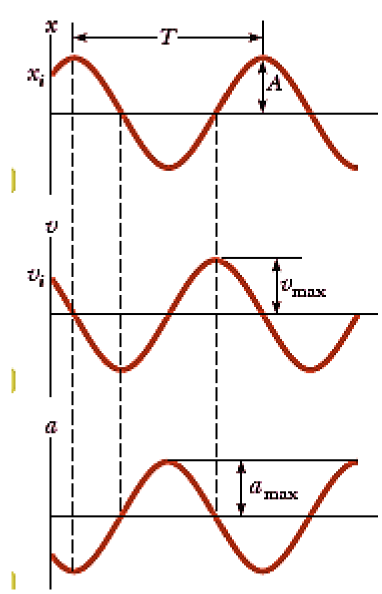
$$v = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$$

$$a = \frac{d^2x}{dt^2} = -\omega^2 A \cos(\omega t + \phi)$$

Therefore, the maximum values of the magnitudes of the velocity and acceleration are

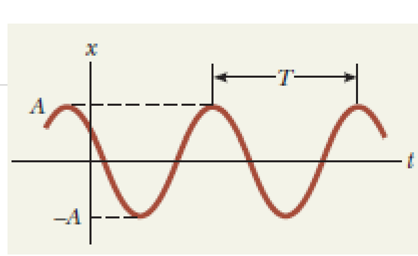
$$v_{\max} = \omega A = \sqrt{\frac{k}{m}}A$$

$$a_{\max} = \omega^2 A = \frac{k}{m}A$$



Graphical representation of simple harmonic motion

Example: - A 200g block connected to a light spring for which the force constant is 5N/m is free to oscillate on a frictionless, horizontal surface. The block is



displaced 5 cm from equilibrium and released from rest as in Figure.

(A) Find the period of its motion.

Energy of the Simple Harmonic Oscillator

Kinetic energy of a simple harmonic oscillator is given by:

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi)$$

Potential energy of a simple harmonic oscillator

$$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \cos^2(\omega t + \phi)$$

The total mechanical energy of the simple harmonic oscillator can be given as

$$E = K + U = \frac{1}{2}kA^2[\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)]$$

$$E = \frac{1}{2}kA^2$$

That is, the total mechanical energy of a simple harmonic oscillator is a constant of the motion and is proportional to the square of the amplitude.

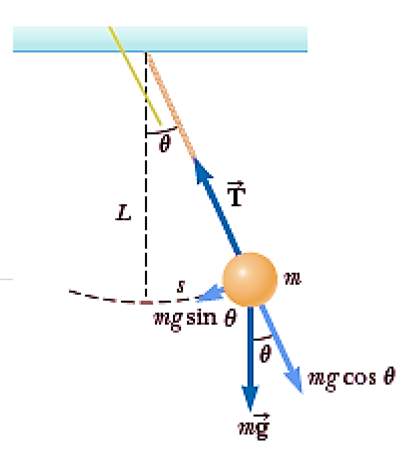
Finally, we can obtain the velocity of the block at an arbitrary position by expressing the total energy of the system at some arbitrary position x as

$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

$$v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)} = \pm\omega\sqrt{A^2 - x^2}$$

The Simple Pendulum

The simple pendulum is another mechanical system that exhibits periodic motion. It consists of a particle-like bob of mass m suspended by a light string of length L that is fixed at the upper end as shown in Figure. The motion occurs in the vertical plane and is driven by the gravitational force. We shall show that, provided the



angle u is small (less than about 108), the motion is very close to that of a simple harmonic oscillator.

$$-mg\sin\theta = m\frac{d^2s}{dt^2}$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta$$

Therefore, for small angles, the equation of motion becomes

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta$$

The angular frequency ω and the period of the motion are given by:

$$\omega = \sqrt{\frac{g}{L}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

The simple pendulum can be used as a timekeeper because its period depends only on its length ***L*** and the local value of **g**.

Example

1. If a simple pendulum oscillates with small amplitude and its length is doubled, what happens to the frequency of its motion? (a) It doubles. (b) It becomes $\sqrt{2}$ times as large. (c) It becomes half as large. (d) It becomes $1/\sqrt{2}$ times as large. (e) It remains the same.
2. An object–spring system moving with simple harmonic motion has amplitude A. When the kinetic energy of the object equals twice the potential energy stored in the spring, what is the position x of the object? (a) A (b) $\frac{1}{3}A$ (c) $\frac{A}{\sqrt{3}}$ (d) 0 (e) none
3. An object of mass 0.4 kg, hanging from a spring with a spring constant of 8N/m, is set into an up-and down simple harmonic motion. What is the magnitude of the acceleration of the object when it is at its maximum displacement of 0.1m? (a) zero (b) 0.45 m/s^2 (c) 1m/s^2 (d) 2m/s^2 (e) 2.4 m/s^2

Wave and its characteristics

Wave motion *is the transfer of energy through space without the accompanying transfer of matter. In the list of energy transferring mechanisms the two mechanisms depend on mechanical waves and electromagnetic radiation.*

The two main types of wave are ***mechanical*** waves and ***electromagnetic waves***. Mechanical wave is the propagation of a disturbance through a medium, example: - water waves, sound waves, waves on a string. Electromagnetic waves do not require a medium to propagate; some examples of electromagnetic waves are visible light, radio waves, television signals, and x-rays.

The two types of mechanical wave are: transverse wave and longitudinal wave.

A ***transverse wave*** is one in which the elements of the medium move in a direction perpendicular to the direction of propagation. An example is a wave on a taut string.

A ***longitudinal wave*** is one in which the elements of the medium move in a direction parallel to the direction of propagation. Sound waves in fluids are longitudinal.

Electro-magnetism

ELECTROSTATICS

The fundamental forces of nature are: - gravitational force, electromagnetic force, weak nuclear force and strong nuclear force.

The electromagnetic force between charged particles is one of the fundamental forces of nature.

Charge

In a series of simple experiments, it was found that there are two kinds of electric charges, which were given the names ***positive*** and ***negative*** by Benjamin Franklin (1706–1790). We identify negative charge as that type possessed by electrons and positive charge as that possessed by protons.

Properties of Electric Charges

- There are two kinds of charges in nature; charges of opposite sign attract one another and charges of the same sign repel one another.
- **Total charge in an isolated system is conserved.**

Charge can be transferred between different materials using different charge transferring mechanisms **i.e.** conduction, induction, rubbing.

- **Charge is quantized.**

In 1909, Robert Millikan (1868–1953) discovered that electric charge always occurs as some integral multiple of a fundamental amount of charge $\boldsymbol{e}$. The electric charge q is said to be quantized, where q is the standard symbol used for charge as a variable. That is, electric charge exists as discrete "packets," and we can write $q = Ne$, where N is some integer.

NB *Electrical conductors are materials in which some of the electrons are free electrons1 that are not bound to atoms and can move relatively freely through the material; electrical insulators are materials in which all electrons are bound to atoms and cannot move freely through the material.*

Coulomb's Law

Charles Coulomb (1736–1806) measured the magnitudes of the electric forces between charged objects using the torsion balance. Coulomb confirmed that the electric force between two small charged spheres

- Is proportional to the inverse square of their separation distance r and directed along the line joining them;
- Is proportional to the product of the charges q_1 and q_2 on the two particles;
- is attractive if the charges are of opposite sign and repulsive if the charges have the same sign;

$$F_e = k_e \frac{|q_1||q_2|}{r^2}$$

Where, ke is a constant called the Coulomb constant. The SI unit of charge is the coulomb (C).

$$k_e = 8.987\,5 \times 10^9 \ \mathrm{N \cdot m^2/C^2}$$

This constant is also written in the form

$$k_e = \frac{1}{4\pi\epsilon_o}$$

Where, the constant ϵ_o (lowercase Greek epsilon) is known as the permittivity of free space and has the value

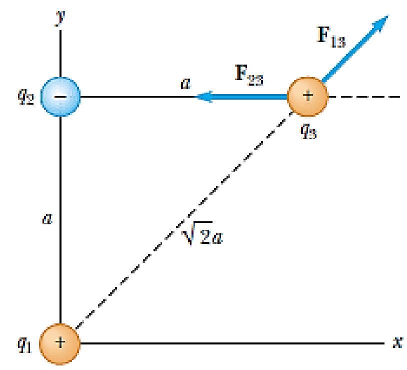
$$\epsilon_0 = 8.854\,2 \times 10^{-12} \ \mathrm{C^2/N \cdot m^2}$$

Charge and Mass of the Electron, Proton, and Neutron

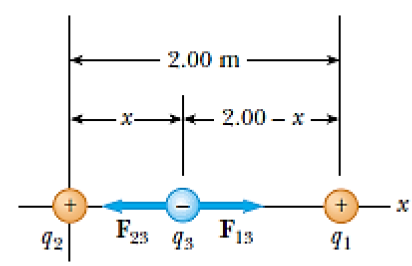
Particle	Charge (C)	Mass (kg)
Electron (e)	$-1.602\,191\,7 \times 10^{-19}$	$9.109\,5 \times 10^{-31}$
Proton (p)	$+1.602\,191\,7 \times 10^{-19}$	$1.672\,61 \times 10^{-27}$
Neutron (n)	0	$1.674\,92 \times 10^{-27}$

Example 1: The electron and proton of a hydrogen atom are separated (on the average) by a distance of approximately $5.3 \times 10^{-11}\,m$. Find the magnitudes of the electric force and the gravitational force between the two particles.

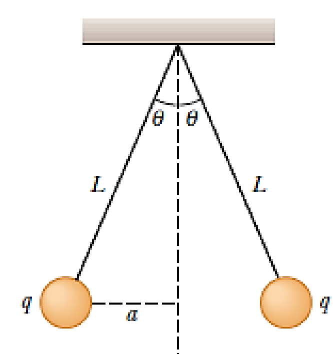
Example2: Consider three point charges of magnitudes $q_1 = q_3 = 5\mu C$ & $q_2 = -2\mu C$ located at the corners of a right triangle as shown in the figure below. Find the resultant force exerted on q_3.



Example 3: Three point charges lie along the x axis as shown in Figure below. The positive charge $q_1 = 15\mu C$ is at $x = 2m$, the positive charge $q_2 = 6\mu C$ is at the origin, and the resultant force acting on q_3 is zero. What is the x coordinate of q_3?



Example 4: Two identical small charged spheres, each having a mass of $3 \times 10^{-2} kg$, hang in equilibrium as shown in Figure below. The length of each string is 0.15m, and the angle θ is 5°. Find the magnitude of the charge on each sphere.



The Electric Field

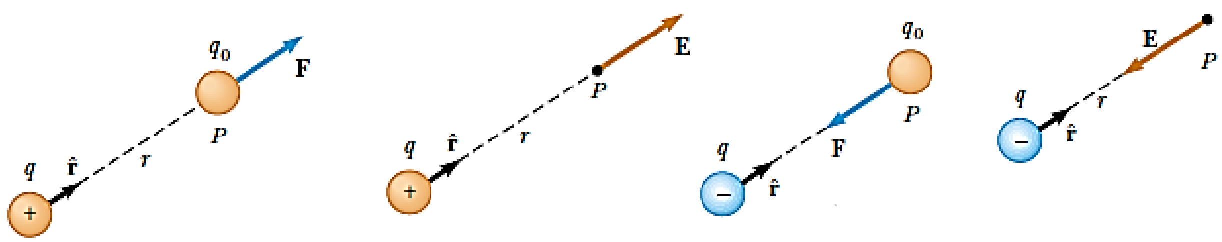
The electric field vector E at a point in space is defined as the electric force F*e* acting on a positive test charge q_o placed at that point divided by the test charge:

$$\vec{E} = \frac{F_e}{q_o}$$

The vector E has the **SI** units of newton per coulomb (N/C).

NB $\vec{\mathbf{E}}$ is the field produced by some charge or charge distribution *separate from* the test charge—it is not the field produced by the test charge itself. Also, note that the existence of an electric field is a property of its source—the presence of the test charge is not necessary for the field to exist. The test charge serves as a *detector* of the electric field.

Note that, an electric field exists at a point if a test charge at that point experiences an electric force.
To determine the direction of an electric field, consider a point charge q as a source charge. This charge creates an electric field at all points in space surrounding it. A test charge q_0 is placed at point P, a distance r from the source charge, as in Figure below.



According to Coulomb's law, the force exerted by q on the test charge is:

$$\mathbf{F}_e = k_e \frac{q q_0}{r^2} \hat{\mathbf{r}}$$

Because the electric field at P, the position of the test charge, is defined by $E = F_e/q_o$, we find that at P, the electric field created by q is:

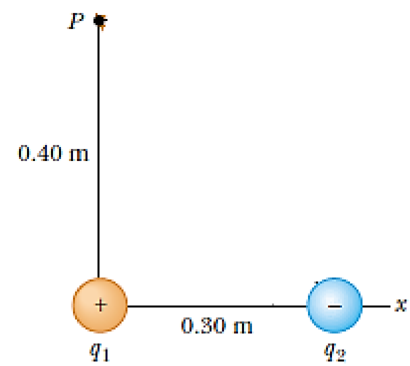
$$\mathbf{E} = k_e \frac{q}{r^2} \hat{\mathbf{r}}$$

Electric field due to a finite number of point charges

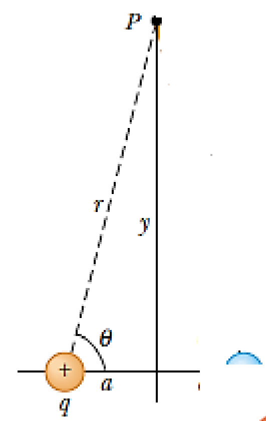
At any point P, the total electric field due to a group of source charges equals the vector sum of the electric fields of all the charges.

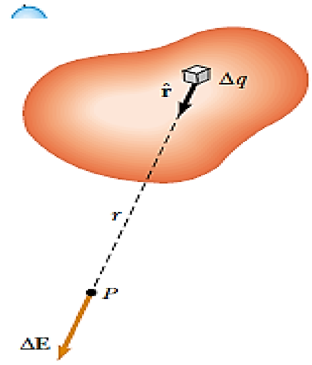
$$\mathbf{E} = k_e \sum_i \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i$$

Example 1: A charge $q_1 = 7\mu C$ is located at the origin, and a second charge $q_2 = -5\mu C$ is located on the x axis, 0.3m from the origin. Find the electric field at the point P, which has coordinates (0, 0.4) m.



Example 1: An electric dipole is defined as a positive charge q and a negative charge $-q$ separated by a distance $2a$. For the dipole shown in Figure below, find the electric field $\vec{E}$ at P due to the dipole, where P is a distance $y \gg a$ from the origin.





Electric Field of a Continuous Charge Distribution

The electric field at P due to a continuous charge distribution is the vector sum of the fields $\Delta\boldsymbol{E}$ due to all the elements Δq of the charge distribution.

$$\Delta \mathbf{E} = k_e \frac{\Delta q}{r^2} \hat{\mathbf{r}}$$

The total electric field at P due to all elements in the charge distribution is approximately

$$\mathbf{E} \approx k_e \sum_i \frac{\Delta q_i}{r_i^2} \hat{\mathbf{r}}_i$$

Where, the index i refers to the i[th] element in the distribution. Because the charge distribution is modeled as continuous, the total field at P in the limit $\Delta q \to 0$ is

$$\mathbf{E} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_i^2} \hat{\mathbf{r}}_i = k_e \int \frac{dq}{r^2} \hat{\mathbf{r}}$$

Charge density

Volume charge density

If a charge Q is uniformly distributed throughout a volume V, the volume charge density is defined by:

$$\rho = \frac{Q}{V}$$

Surface charge density

If a charge Q is uniformly distributed on a surface of area A, the surface charge density (lowercase Greek sigma) is defined by:

$$\sigma = \frac{Q}{A}$$

Linear charge density

If a charge Q is uniformly distributed along a line of length L, the linear charge density is defined by:

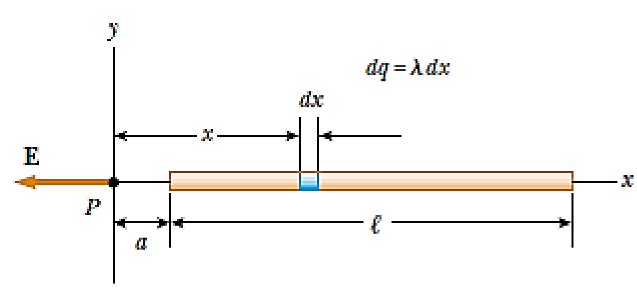
$$\lambda = \frac{Q}{L}$$

If the charge is non-uniformly distributed over a volume, surface, or line, the amounts of charge dq in a small volume, surface, or length element are:

$$dq = \rho\, dV \qquad dq = \sigma\, dA \qquad dq = \lambda\, d\ell$$

Example: A rod of length L has a uniform positive charge per unit length λ and a total charge Q. Calculate the electric field at a point P that is located along the long axis of the rod and a distance ***a*** from one end.

Solution



$$dE = k_e \frac{dq}{x^2} = k_e \frac{\lambda\, dx}{x^2}$$

$$E = \int_a^{\ell + a} k_e \lambda \frac{dx}{x^2}$$

$$E = k_e \lambda \int_a^{\ell + a} \frac{dx}{x^2} = k_e \lambda \left[-\frac{1}{x} \right]_a^{\ell + a}$$

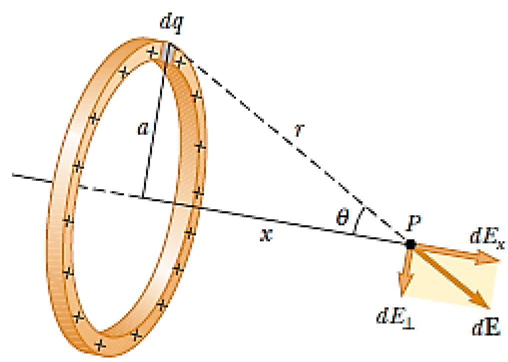
$$= k_e \lambda \left(\frac{1}{a} - \frac{1}{\ell + a} \right)$$

$$= \frac{k_e Q}{a(\ell + a)}$$

Example: A ring of radius a carries a uniformly distributed positive total charge Q. Calculate the electric field due to the ring at a point P lying a distance x from its center along the central axis perpendicular to the plane of the ring:

Solution

The magnitude of the electric field at P due to the segment of charge dq is



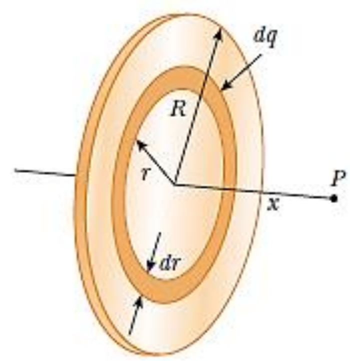
$$dE = k_e \frac{dq}{r^2}$$

$$dE_x = dE\cos\theta = \left(k_e \frac{dq}{r^2}\right)\frac{x}{r} = \frac{k_e x}{(x^2 + a^2)^{3/2}}\, dq$$

$$E_x = \int \frac{k_e x}{(x^2 + a^2)^{3/2}}\, dq = \frac{k_e x}{(x^2 + a^2)^{3/2}} \int dq$$

$$= \frac{k_e x}{(x^2 + a^2)^{3/2}}\, Q$$

Example: A disk of radius R has a uniform surface charge density. Calculate the electric field at a point P that lies along the central perpendicular axis of the disk and a distance x from the center of the disk:



Solution:

The ring of radius r and width dr shown in Figure has a surface area equal to $2\pi r\, dr$.

$$dE_x = \frac{k_e x}{(x^2 + r^2)^{3/2}} (2\pi\sigma r\, dr)$$

$$E_x = k_e x\pi\sigma \int_0^R \frac{2r\, dr}{(x^2 + r^2)^{3/2}}$$

$$= k_e x\pi\sigma \int_0^R (x^2 + r^2)^{-3/2}\, d(r^2)$$

$$= k_e x\pi\sigma \left[\frac{(x^2 + r^2)^{-1/2}}{-1/2}\right]_0^R$$

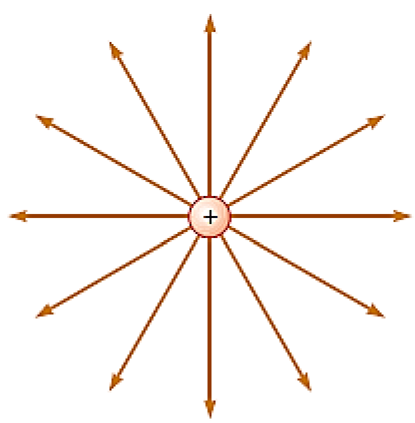
$$= 2\pi k_e \sigma \left(1 - \frac{x}{(x^2 + R^2)^{1/2}}\right)$$

Electric Field Lines

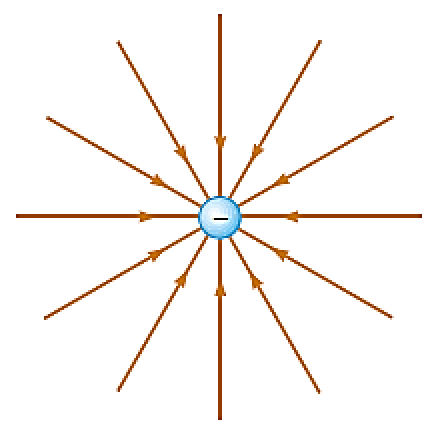
Electric field lines describe an electric field in any region of space. The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of E in that region.

Properties of electric field lines

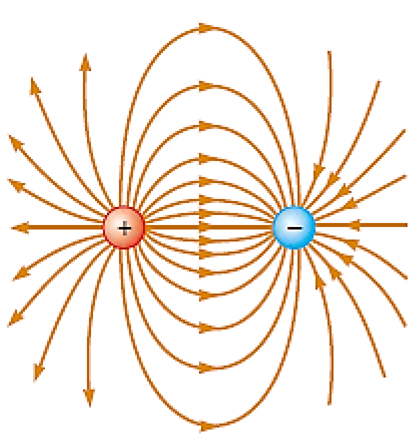
i. The electric field lines due to a positive source charge indicate radially out ward. Whereas, the electric field lines due to a negative source charge are radially in ward.

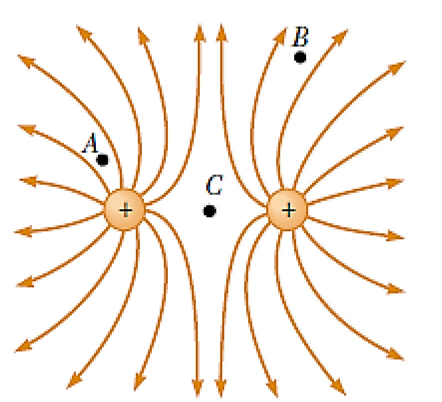


radially out ward



radially in ward





ii. No two electric field lines will cross each other. (are parallel)

iii. The number of field lines in a given space indicates the strength of the electric field at that position (the field lines are close together where the electric field is strong and far apart where the field is weak).

Motion of Charged Particles in a Uniform Electric Field

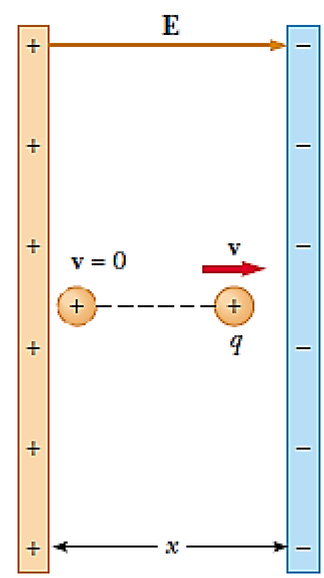
When a particle of charge **q** and mass **m** is placed in an electric field E, the electric force exerted on the charge is **qE**. If this is the only force exerted on the particle, it must be the net force and causes the particle to accelerate according to Newton's second law. Thus,

$$\mathbf{F}_e = q\mathbf{E} = m\mathbf{a}$$

The acceleration of the particle is therefore

$$a = \frac{qE}{m}$$

If E is uniform (that is, constant in magnitude and direction), then the acceleration is constant. If the particle has a positive charge, its acceleration is in the direction of the electric field. If the particle has a negative charge, its acceleration is in the direction opposite the electric field.



Example: A positive point charge q of mass m is released from rest in a uniform electric field E directed along the x axis, as shown in Figure below. Find an expression for the position, velocity and work done due to its motion.

Solution

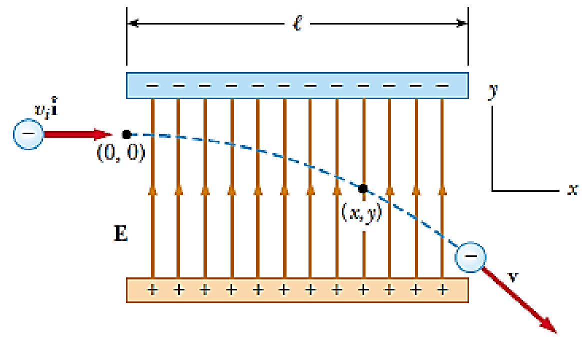
$$x_f = \tfrac{1}{2}at^2 = \frac{qE}{2m}t^2$$

$$v_f = at = \frac{qE}{m}t$$

$$v_f^{\,2} = 2ax_f = \left(\frac{2qE}{m}\right)x_f$$

$$W = \Delta K = \tfrac{1}{2}mv_f^2 = \tfrac{1}{2}m\left(\frac{2qE}{m}\right)\Delta x = qE\Delta x$$

Example: An electron enters the region of a uniform electric field as shown in Figure below, with $v_i = 3 \times 10^6$m/s and $E = 200$ N/C. The horizontal length of the plates is $L = 0.1m$.

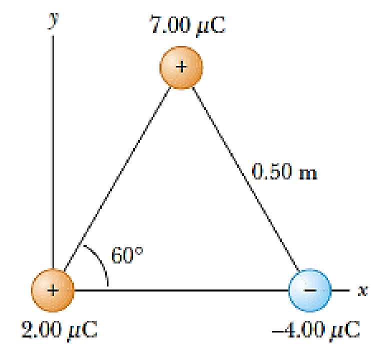


(A) Find the acceleration of the electron while it is in the electric field.

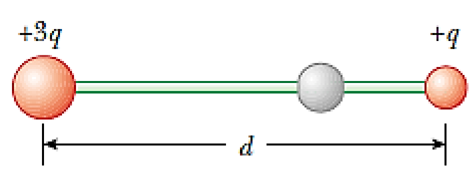
(B) If the electron enters the field at time $t = 0$, find the time at which it leaves the field.

Exercise

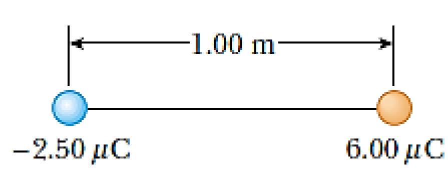
1. Three point charges are located at the corners of an equilateral triangle as shown in Figure below. Calculate the resultant electric force on the $7\mu C$ charge.



2. Two small beads having positive charges $3q$ and q are fixed at the opposite ends of a horizontal, insulating rod, extending from the origin to the point $x = d$. As shown in Figure below, a third small charged bead is free to slide on the rod. At what position is the third bead in equilibrium? Can it be in stable equilibrium?



3. Determine the point (other than infinity) at which the electric field is zero.



4. Consider an infinite number of identical charges (each of charge *q*) placed along the *x* axis at distances *a*, 2*a*, 3*a*, 4*a*, . . . , from the origin. What is the electric field at the origin due to this distribution? *Suggestion:* Use the fact that

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \frac{\pi^2}{6}$$

Electric potential

For an infinitesimal displacement $\boldsymbol{ds}$ of a charge, the work done by the electric field on the charge is $W = q_o Eds$. As this amount of work is done by the field, the potential energy of the charge-field system is changed by an amount $\mathrm{dU} = -\mathrm{qE.ds}$.

For a finite displacement of the charge from point *A* to point *B*, the change in potential energy of the system

$$\Delta U = U_B - U_A = -q_o \int_A^B E.dS$$

The potential difference $\Delta V = V_A - V_B$between two points *A* and *B* in an electric field is defined as the change in potential energy of the system when a test charge is moved between the points divided by the test charge **q_0**:

$$\Delta V \equiv \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

The potential energy per unit charge U/q_o is independent of the value of **q_o** and has a value at every point in an electric field and is called the electric potential.

Electric potential is a scalar characteristic of an electric field, independent of any charges that may be placed in the field.

Because electric potential is a measure of potential energy per unit charge, the SI unit of both electric potential and potential difference is joules per coulomb, which is defined as a volt (V):

Electric Potential and Potential Energy Due to Point Charges

An isolated positive point charge *q* produces an electric field that is directed radially outward from the charge. To find the electric potential at a point located a distance *r* from the charge, we begin with the general expression for potential difference:

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

$$V_B - V_A = -k_e q \int_{r_A}^{r_B} \frac{dr}{r^2} = \frac{k_e q}{r} \Bigg]_{r_A}^{r_B}$$

$$V_B - V_A = k_e q \left[\frac{1}{r_B} - \frac{1}{r_A} \right]$$

The electric potential created by a point charge at any distance r from the charge is

$$V = k_e \frac{q}{r}$$

The total electric potential at some point P due to several point charges is the sum of the potentials due to the individual charges.

$$V = k_e \sum_i \frac{q_i}{r_i}$$

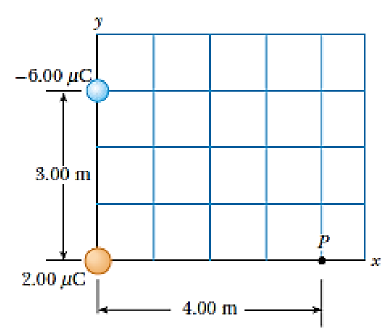
Example: A charge $q_1 = 2\mu C$ is located at the origin, and a charge $q_2 = -6\mu C$ is located at (0, 3) m, as shown in Figure below.

(A) Find the total electric potential due to these charges at the point P, whose coordinates are (4, 0) m.

$$V_P = k_e \left(\frac{q_1}{r_1} + \frac{q_2}{r_2} \right)$$

$$V_P = (8.99 \times 10^9 \ \mathrm{N \cdot m^2/C^2}) \times \left(\frac{2.00 \times 10^{-6} \ \mathrm{C}}{4.00 \ \mathrm{m}} - \frac{6.00 \times 10^{-6} \ \mathrm{C}}{5.00 \ \mathrm{m}} \right)$$

$$= -6.29 \times 10^3 \ \mathrm{V}$$

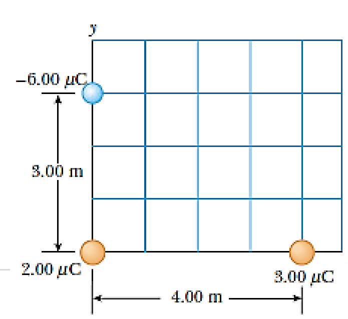


(B) Find the change in potential energy of the system of two charges plus a charge $q_3 = 3\mu C$ as the latter charge moves from infinity to point P

$$U = k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$

$$= (8.99 \times 10^9 \ \mathrm{N \cdot m^2/C^2}) \times \left(\frac{(2.00 \times 10^{-6} \ \mathrm{C})(-6.00 \times 10^{-6} \ \mathrm{C})}{3.00 \ \mathrm{m}} + \frac{(2.00 \times 10^{-6} \ \mathrm{C})(3.00 \times 10^{-6} \ \mathrm{C})}{4.00 \ \mathrm{m}} + \frac{(3.00 \times 10^{-6} \ \mathrm{C})(-6.00 \times 10^{-6} \ \mathrm{C})}{5.00 \ \mathrm{m}} \right)$$

$$= -5.48 \times 10^{-2} \ \mathrm{J}$$



Obtaining the Value of the Electric Field

from the Electric Potential

The potential difference dV between two points a distance ds apart as

$$dV = -E, dS$$

If the electric field has only one component *Ex*, then $E, dS = E_x. dx.$ Therefore,

$$E_x = -\frac{dV}{dx}$$

In general, the electric potential is a function of all three spatial coordinates. If $V(r)$ is given in terms of the Cartesian coordinates, the electric field components *Ex*, *Ey*, and *Ez* can readily be found from $V(x, y, z)$ as the partial derivatives:

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

Example: if $V(x, y, z) = 3x^2y + y^2 + yz,$ then find an expression for the electric field

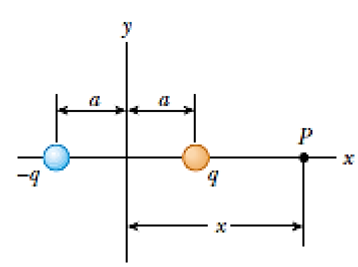
In vector notation, E is often written in Cartesian coordinate systems as

$$\mathbf{E} = -\nabla V = -\left(\hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z}\right)V$$

Example:

An electric dipole consists of two charges of equal magnitude and opposite sign separated by a distance $2a$, as shown in Figure below. The dipole is along the x axis and is centered at the origin.

(A) Calculate the electric potential at point *P*.



$$V = k_e \sum \frac{q_i}{r_i} = k_e\left(\frac{q}{x-a} - \frac{q}{x+a}\right) = \frac{2k_e qa}{x^2 - a^2}$$

(B) Calculate V and Ex at a point far from the dipole.

If point P is far from the dipole, such that $x \gg a,$ then $\boldsymbol{a^2}$ can be neglected in the term $x^2 - a^2$ and V becomes

$$V \approx \frac{2k_e qa}{x^2} \qquad (x \gg a)$$

We can calculate the magnitude of the electric field at a point far from the dipole:

$$E_x = -\frac{dV}{dx} = \frac{4k_e qa}{x^3} \qquad (x \gg a)$$

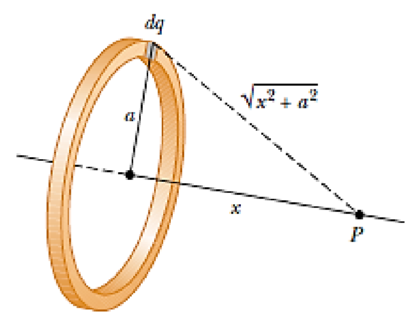
Electric Potential Due to Continuous Charge Distributions

The electric potential dV at some point P due to the charge element dq is

$$dV = k_e \frac{dq}{r}$$

$$V = k_e \int \frac{dq}{r}$$

Example: **(A)** Find an expression for the electric potential at a point P located on the perpendicular central axis of a uniformly charged ring of radius a and total charge Q.



$$V = k_e \int \frac{dq}{r} = k_e \int \frac{dq}{\sqrt{x^2 + a^2}}$$

$$V = \frac{k_e}{\sqrt{x^2 + a^2}} \int dq = \frac{k_e Q}{\sqrt{x^2 + a^2}}$$

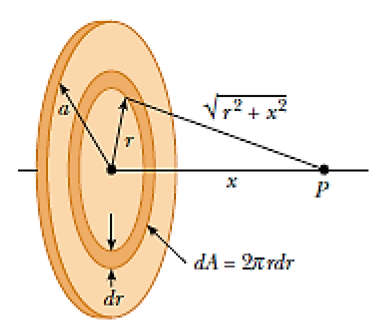
(B) Find an expression for the magnitude of the electric field at point P.

$$E_x = -\frac{dV}{dx} = -k_e Q \frac{d}{dx}(x^2 + a^2)^{-1/2}$$

$$= -k_e Q(-\tfrac{1}{2})(x^2 + a^2)^{-3/2}(2x)$$

$$E_x = \frac{k_e Q x}{(x^2 + a^2)^{3/2}}$$

Example: A uniformly charged disk has radius **a** and surface charge density σ. Find



(A) The electric potential and

$$dV = \frac{k_e dq}{\sqrt{r^2 + x^2}} = \frac{k_e \sigma 2\pi r\, dr}{\sqrt{r^2 + x^2}}$$

$$V = \pi k_e \sigma \int_0^a \frac{2r\, dr}{\sqrt{r^2 + x^2}} = \pi k_e \sigma \int_0^a (r^2 + x^2)^{-1/2}\, 2r\, dr$$

$$V = 2\pi k_e \sigma \left[(x^2 + a^2)^{1/2} - x\right]$$

(B) The magnitude of the electric field along the perpendicular central axis of the disk.

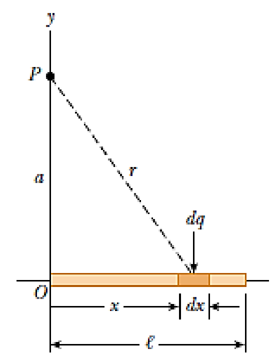
$$E_x = -\frac{dV}{dx} = 2\pi k_e \sigma \left(1 - \frac{x}{\sqrt{x^2 + a^2}}\right)$$

Example: A rod of length l located along the x axis has a total charge Q and a uniform linear charge density $\lambda = Q/l$. Find the electric potential at a point P located on the y axis a distance **a** from the origin

We can express the potential at point P due to this element as

$$dV = k_e \frac{dq}{r} = k_e \frac{\lambda\, dx}{\sqrt{x^2 + a^2}}$$

$$V = k_e \lambda \int_0^{\ell} \frac{dx}{\sqrt{x^2 + a^2}} = k_e \frac{Q}{\ell} \int_0^{\ell} \frac{dx}{\sqrt{x^2 + a^2}}$$



This integral has the following value

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

$$V = \frac{k_e Q}{\ell} \ln\left(\frac{\ell + \sqrt{\ell^2 + a^2}}{a}\right)$$

Example: An insulating solid sphere of radius R has a uniform positive volume charge density and total charge Q.

(A) Find the electric potential at a point outside the sphere, that is, for $r > R$. Take the potential to be zero at $r = \infty$

We found that the magnitude of the electric field outside a uniformly charged sphere of radius R is

$$E_r = k_e \frac{Q}{r^2} \quad (\text{for } r > R)$$

$$V_B - V_A = k_e Q \left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

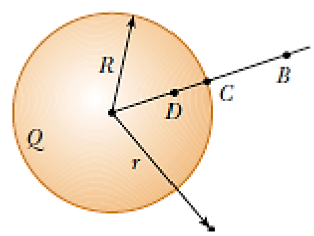
$$V_B - 0 = k_e Q \left[\frac{1}{r_B} - 0\right]$$

$$V_B = k_e \frac{Q}{r} \quad (\text{for } r > R)$$

$$V_C = k_e \frac{Q}{R} \qquad (\text{for } r = R)$$

(B) Find the potential at a point inside the sphere, that is, for $r < R$.

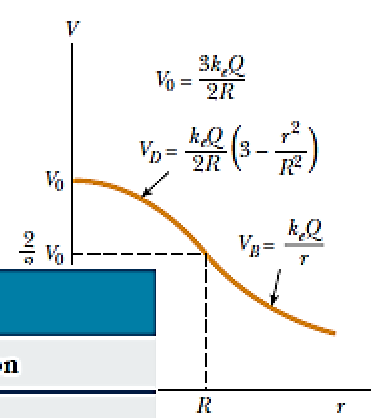
$$E_r = \frac{k_e Q}{R^3} r \qquad (\text{for } r < R)$$



$$V_D - V_C = -\int_R^r E_r\, dr = -\frac{k_e Q}{R^3}\int_R^r r\, dr = \frac{k_e Q}{2R^3}(R^2 - r^2)$$

$$V_C = k_e Q/R$$

$$V_D = \frac{k_e Q}{2R}\left(3 - \frac{r^2}{R^2}\right)$$



Electric Potential Due to Various Charge Distributions

Charge Distribution	Electric Potential	Location
Uniformly charged ring of radius a	$V = k_e \frac{Q}{\sqrt{x^2 + a^2}}$	Along perpendicular central axis of ring, distance x from ring center
Uniformly charged disk of radius a	$V = 2\pi k_e \sigma[(x^2 + a^2)^{1/2} - x]$	Along perpendicular central axis of disk, distance x from disk center
Uniformly charged, *insulating* solid sphere of radius R and total charge Q	$V = k_e \frac{Q}{r}$	$r \geq R$
	$V = \frac{k_e Q}{2R}\left(3 - \frac{r^2}{R^2}\right)$	$r < R$
Isolated *conducting* sphere of radius R and total charge Q	$V = k_e \frac{Q}{r}$	$r > R$
	$V = k_e \frac{Q}{R}$	$r \leq R$

Direct current circuits

Current and Resistance

The time rate at which charge flows through the area is defined as the current *I*. The direction of the current is the direction in which positive charges flow when free to do so.

Whenever there is a net flow of charge through some region, an electric current is said to exist.

Definition: -

The current is the rate at which charge flows through this surface.

$$\boldsymbol{I = \frac{\Delta Q}{\Delta t}}$$

If the rate at which charge flows varies in time, then the current varies in time; we define the instantaneous current *I* as the differential limit of average current:

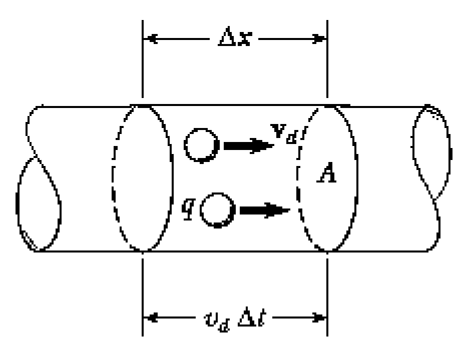
$$I = \frac{dq}{dt}$$

The SI unit of current is the ampere (A): $1A = 1C/s$

Microscopic Model of Current

Consider the current in a conductor of cross-sectional area *A*. The volume of a section of the conductor of length Δx shown in figure below is $A\,\Delta x$.

If *n* represents the number of mobile charge carriers per unit volume (the charge carrier density), the number of carriers is $nA\Delta x$. Therefore, the total charge ΔQ in this section is



$\Delta Q = nA\Delta xq$

Where, q is the charge on each carrier. If the carriers move with a speed v_d, the displacement they experience in the *x* direction in a time interval is $\Delta x = v_d\boldsymbol{\Delta t}$

$$\Delta Q = (nAv_d\,\Delta t)\,q$$

Then

$$I_{av} = \frac{\Delta Q}{\Delta t} = nqv_dA$$

The speed of the charge carriers v_d is an average speed called the **drift speed**.

Resistance

Consider a conductor of cross-sectional area A carrying a current I. The current density J in the conductor is defined as the current per unit area. Because the current $I = nqv_dA$, the current density is**:**

$$J \equiv \frac{I}{A} = nqv_d$$

Where J has SI units of A/m^2

Ohm's Law

The ratio of the current density to the electric field is a constant & that is independent of the electric field producing the current.

$$J = \sigma E$$

Where, σ is conductivity of the conductor.

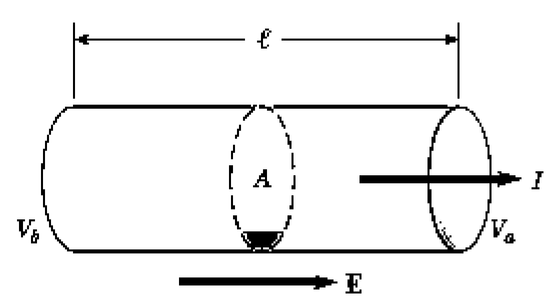
Materials that obey Ohm's law and hence demonstrate this simple relationship between E and J are said to be *ohmic.*

Consider a uniform conductor of length l and cross-sectional area A. A potential difference $\Delta V = V_b - V_a$ maintained across the conductor sets up an electric field **E**, and this field produces a current I that is proportional to the potential difference.

$\Delta V = El$

Therefore, we can express the magnitude of the current density in the wire as

$$J = \sigma E = \sigma \frac{\Delta V}{\ell}$$



We can write the potential difference as

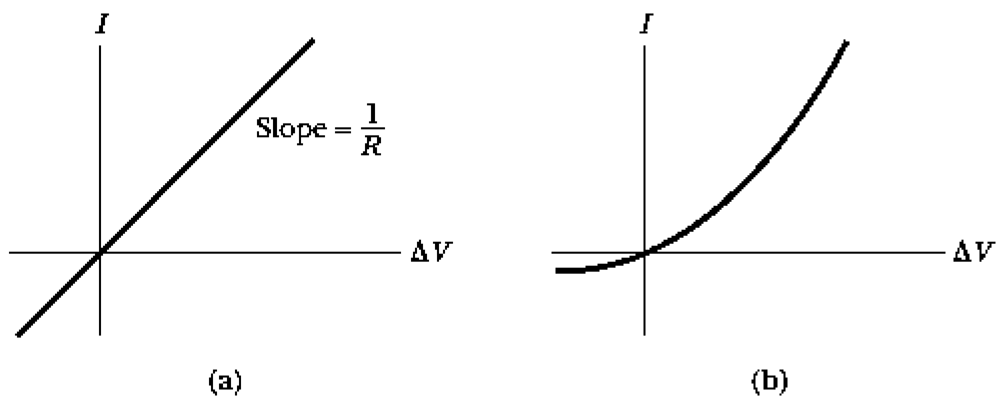
$$\Delta V = \frac{\ell}{\sigma} J = \left(\frac{\ell}{\sigma A}\right) I = RI$$

The quantity $R = l/\sigma A$ is called resistance of the conductor. Thus,

$$R = \frac{\Delta V}{I}$$

The inverse of conductivity is resistivity $\rho = \frac{1}{\sigma}$. Hence,

$$R = \frac{\rho l}{A}$$



(a)The current–potential difference curve for an ohmic material;

(b) A nonlinear current–potential difference curve for a junction diode. This device does not obey Ohm's law.

Exercise

(A) Calculate the resistance per unit length of a Nichrome wire, which has a radius of 0.321 mm. $[1.5x10^{-6}\Omega.m]$

$$\boldsymbol{Ans: 4.6\Omega/m}$$

(B) If a potential difference of 10V is maintained across a 1m length of the Nichrome wire, what is the current in the wire?

Ans: 2.2A

Electrical power

If a potential difference ΔV is maintained across a circuit element, the power, or rate at which energy is supplied to the element, is

$$\boldsymbol{P = I\Delta V = I^2R}$$

The energy delivered to a resistor by electrical transmission appears in the form of internal energy in the resistor.

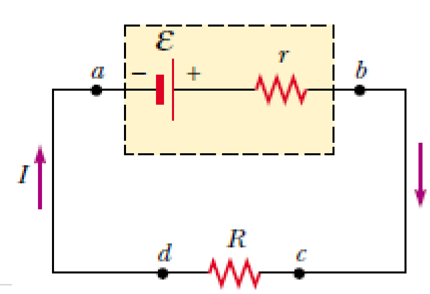
Exercise

A 10V battery is connected to a 120Ω resistor. Ignoring the internal resistance of the battery, calculate the power delivered to the resistor.

Electromotive Force

The emf of a battery is the maximum possible voltage that the battery can provide between its terminals.

Because a real battery is made of matter, there is resistance to the flow of charge within the battery. This resistance is called ***internal resistance*** *r*.



$$\Delta V = \varepsilon - Ir$$

Where, $\Delta V = IR$; R is the load resistance & r internal resistance

Exercise:

A battery has an emf of 12V and an internal resistance of 0.05Ω. Its terminals are connected to a load resistance of 3Ω.

(A) Find the current in the circuit and the terminal voltage of the battery.

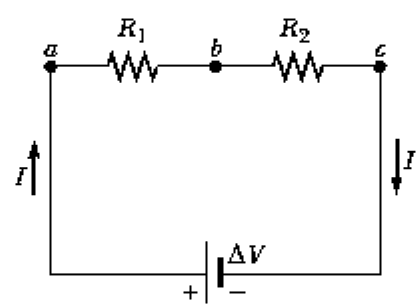
(B) Calculate the power delivered to the load resistor, the power delivered to the internal resistance of the battery, and the power delivered by the battery.

Resistors combination

Resistors in Series

For a series combination of two resistors, the currents are the same in both resistors because the amount of charge that passes through R_1 must also pass through R_2 in the same time interval.

$$\Delta V = IR_1 + IR_2 = I(R_1 + R_2)$$

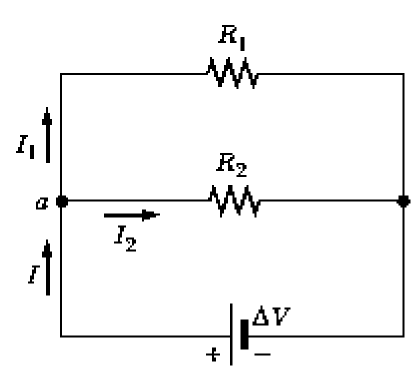


The equivalent resistance of a series connection of resistors is the numerical sum of the individual resistances and is always greater that any individual resistance.

$$R_{eq} = R_1 + R_2 + R_3 + \cdots$$

Resistors in parallel

When resistors are connected in parallel the potential differences across the resistors is the same.



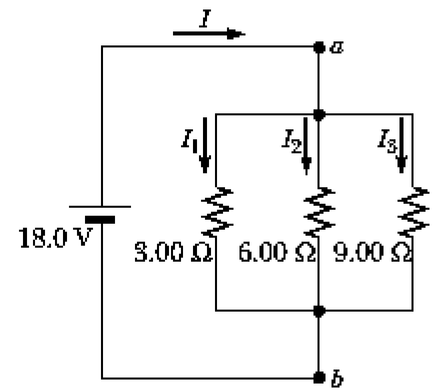
The equivalent resistance of two or more resistors connected in parallel is equal to the sum of the inverses of the individual resistances. Furthermore, the equivalent resistance is always less than the smallest resistance in the group.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

Exercise: - Three resistors are connected in parallel as shown in Figure below. A potential difference of 18 V is maintained between points *a and b*.

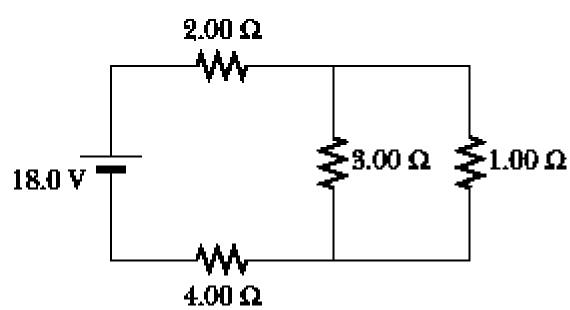
a. Find the current in each resistor and

b. Power across each resistor

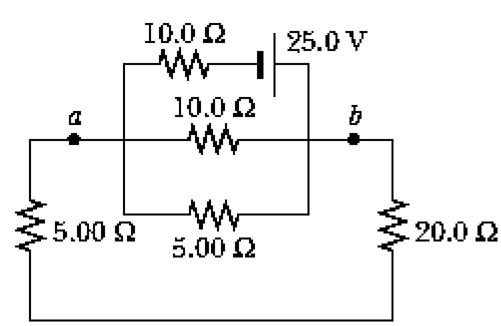


Exercise: -When two unknown resistors are connected in series with a battery, the battery delivers 225 W and carries a total current of 5A. For the same total current, 50W is delivered when the resistors are connected in parallel. Determine the values of the two resistors.

Exercise: -Calculate the power delivered to each resistor in the circuit shown in Figure below.



Exercise: -Consider the circuit shown in Figure below. Find (a) the current in the 20Ω resistor and (b) the potential difference between points *a* and *b*.



Kirchhoff's Rules

The procedure for analyzing more complex circuits is greatly simplified if we use two principles called Kirchhoff's rules:

1. Junction rule. The sum of the currents entering any junction in a circuit must equal the sum of the currents leaving that junction:

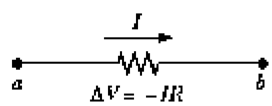
$$\sum I_{in} = \sum I_{out}$$

2. Loop rule. The sum of the potential differences across all elements around any closed circuit loop must be zero:

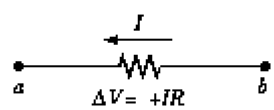
$$\sum_{\substack{\text{closed}\\\text{loop}}} \Delta V = 0$$

When applying Kirchhoff's second rule in practice, we imagine *traveling* around the loop and consider changes in *electric potential*, note the following sign conventions when using the second rule:

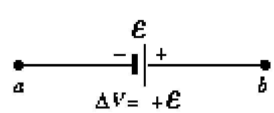
✓ Because charges move from the high-potential end of a resistor toward the low potential end, if a resistor is traversed in the direction of the current, the potential difference across the resistor is $\Delta V = -IR$.



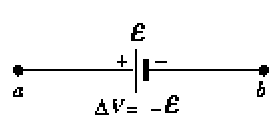
✓ If a resistor is traversed in the direction *opposite* the current, the potential difference across the resistor is ΔV = -IR .



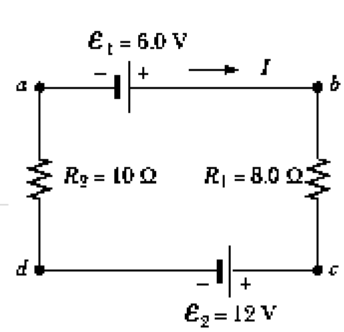
✓ If a source of emf (assumed to have zero internal resistance) is traversed in the direction of the emf (from -ve to +ve), the potential difference is +ve. The emf of the battery increases the electric potential as we move through it in this direction.



✓ If a source of emf (assumed to have zero internal resistance) is traversed in the direction opposite the emf (from +ve to -ve), the potential difference is -ve (Fig. 28.15d). In this case



<u>Exercise</u>: -A single-loop circuit contains two resistors and two batteries, as shown in Figure below. (Neglect the internal resistances of the batteries.)



(A) Find the current in the circuit.

(B) What power is delivered to each resistor? What power is delivered by the 12-V battery?

Exercise: - fine the currents in the circuit shown below